Compton polarizabilities of π-mesons in relativistic Hamiltonian dynamics

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Abstract

The electric and magnetic polarizabilities of the charged pion are studied in the framework of the Poincare-covariant quark. We consider $\pi^{\pm}$-meson as relativistic two-particle system with realistic potential describing the strong interactions between quarks. We use the relativistic Hamiltonian dynamics for description of the relativistic bound system of two spinor quarks and the one-gluon-exchange-plus-linear-confinement potential motivated by QCD with smearing parameter.

Introduction

The calculation of electromagnetic characteristics of bound systems enables to determine not only the quantum numbers of the particles that make up the systems, but also to study the interactions among them. The electric $\bar{\alpha}$ and magnetic polarizabilities $\bar{\beta}$ are fundamental structure parameters characterizing the ability of the hadron to be deformed be external fields one of the characteristics (see, ***V.A. Petrun'kin, Fiz. Elem. Chastis At. Yadra V.12. (1981) P.692. [Sov.J. Part.Nucl.V.12 (1981) P.278]***;
A.I. L'vov, Int.Journ. Mod. Phys. A8 ***(1993). P.5267***).

The generalized (Compton) electric polarizability $\bar{\alpha}$ can be represented as a sum

$$\bar{\alpha} = \alpha_0 + \Delta\alpha, \tag{1}$$

where α_0 is a static electric polarizability and $\Delta\alpha$ is correction term.

The static electir polarizability can be represented as (see ***V.A. Petrun'kin, Fiz. Elem. Chastis At. Yadra V.12. (1981) P.692. [Sov.J. Part.Nucl.V.12 (1981) P.278]***

$$\alpha_0 = \frac{2}{3} \sum_{n \neq 0} \frac{\left| \langle n | \mathbf{D} | 0 \rangle \right|^2}{E_n - E_0} \,, \tag{2}$$

where the operator $\mathbf{D}$ is the internal electric dipole operator.

The term $\Delta\alpha$ for spinless meson P is equal to

$$\Delta\alpha = \frac{e_P^2 \left\langle r_P^2 \right\rangle}{3M_P}, \tag{3}$$

where e_P and M_P are the particle charge and mass and r_P is the electric raduis. This term $\Delta\alpha$ has relativistic nature and can be explained due to a modification of the Thomson scattering of point-like charge by the radius of the scatterer.

The expression for the magnetic polarizability has the form

$$\bar{\beta} = \beta_0 + \Delta\beta \tag{4}$$

with the β_0 static magnetic polarizability

$$beta_0 = \frac{2}{3}\sum_{n\neq 0}\frac{|\langle n|\,\mathbf{M}\,|0\rangle|^2}{E_n - E_0}\ , \tag{5}$$

and with the

$$\Delta\beta = -\frac{e_q^2\langle\psi|\,r_q^2\,|\,\psi\rangle}{6m_q} - \frac{e_Q^2\langle\psi|\,r_Q^2\,|\,\psi\rangle}{6m_Q} - \frac{\langle\psi\,|\,\mathbf{D}^2\,|\,\psi\rangle}{2M}\ , \tag{6}$$

where $\mathbf{M}$ is the internal dipole magnetic operator and $r_{q,Q}$ is the corresponding internal radius vector.

The generalized electric polarizability $\bar{\alpha}$ of $\pi^{\pm}$-mesons has been determined from measurements of the Compton scatterring (see ***Yu. M. Antipov** et al.*, ***Phys. Lett. B* 121 *(1983) P.445; Z. Phys. C* 26 *(1985) P.***

495, ***T. A. Aibergenov*** *et al.*, ***Czech. J. Phys. B* 36 *(1986) P. 948***)
or photon-photon scattering ***Mark II Collaboration, J. Boyer*** *et al.*, ***Phys. Rev. D* 42 *(1990) P.1350;***
D. Babusci *et al.*, ***Phys. Lett. B* 277 *(1992) P.158*** . The results of these experiments giving

$$\bar{\alpha}^{\pi^{\pm}}_{exp} = \begin{cases} 6.8 \pm 1.4\,(\text{stat.}) \pm 1.2\,(\text{syst.})\, \textbf{\textit{Yu. M. Antipov}}\ \textit{et al.} \\ 20 \pm 12\,(\text{stat.}),\ \textbf{\textit{T. A. Aibergenov}}\ \textit{et al.}, \\ 2.2 \pm 1.6\,(\text{stat.} + \text{syst.}),\ \textbf{\textit{Mark II Collaboration}} \end{cases} \tag{7}$$

in units of 10^{-4} Fm^3 . The weighted average of these measurements reads (see ***W. Lucha and F. F. Schöberl, Phys. Lett. B* 544 *(2002) P.380; ArXiv:hep-ph/0204325***)

$$\bar{\alpha}^{\pi^{\pm}}_{exp} = (4.3 \pm 1.2) \times 10^{-4}\ \mathrm{Fm}^3 \tag{8}$$

There are many approaches used for the description of the polarizabilities hadrons as bound states of quarks:

the nonrelativistic quark model

G. Dattoli, G. Matone and D. Prosperi, Lett. Nuovo. Cim. 19 (1977) P.601;

D. Drechsel and A. Russo, Phys. Lett. B 137 (1982) P. 295;

F. Schöberl and H. Leeb, Phys. Lett. B 166 *(1986) P.355*;

M. De Sanctis and D. Prosperi, Nuovo. Cim. A 103 (1990) P.1301;

H. Liebl and G.R. Goldstein, Phys. Lett. B 343 (1995) P.363,

the chiral quark model

R. Weiner and W. Weise, Phys. Lett. 159B (1985) P.85;

N.N. Scoccola and W. Weise, Nucl. Phys. A 517 (1990) P.495;

J. F. Donoghue and B. R. Holstein, Phys. Rev. D 40 *(1989) P.2378*,

the semirelativistic description of quark bound state

W. Lucha and F. F. Schöberl, Phys. Lett. B 544 *(2002) P.380;*

ArXiv:hep-ph/0204325,
model with "confinemented" quarks
M. A. Ivanov and T. Mizutani, Phys. Rev. D **45** ***(1992) P.1580***;
and quark model in quasipotential approach
N.V. Maksimenko and S.G. Shulga, Yad.Fiz.**V.56** ***(1993) P.201***
(more detailed review, see ***A.I. L'vov, Int.Journ. Mod. Phys.*** **A8** ***(1993). P.5267***).

One of the important task is to investigate the relativistic ***V.A. Petrun'kin, [Sov.J. Part.Nucl.V.12 (1981) P.278]***, ***W. Lucha and F. F. Schöberl, Phys. Lett. B*** **544** ***(2002) P.380; ArXiv:hep-ph/0204325***, ***R.N.Lee, A.I. Milstein and M.Schumacher, hep-ph/0101240.***.... In particular, the contributions are expected to eliminate the difference between the predictions in the non-relativistic quark model for proton and neutron.

The relativistic corrections to electric polarizability come from the corrections to wave functions and energies of the bound state, and corrections to electric dipole moment operator. The first relativistic corrections to electromagnetic polarizabilities of bound system with electromagnetic interaction was considered in ***R.N.Lee, A.I. Milstein and M.Schumacher, hep-ph/0101240***. In ***W. Lucha and F. F. Schöberl, Phys. Lett. B*** 544 ***(2002) P.380; ArXiv:hep-ph/0204325*** the electric polarizability of mesons was studied in the framework of a semirelativistic (relativistic kinetic energy and nonrelativistic interaction potential of system and electromagnetic field) description of hadrons as bound states of constituent quarks. But no calculations of polarizabilities were made for hadrons as relativistic bound states with realistic interquark potential.

Aim

Aim of this work is to evaluate Compton polarizabilities of π–mesons as relativistic two-particles system with realistic potential describing the strong interactions between quarks.

Constituents of task:

Polarizabilities of relativistic bound system

Task 1.

Description of relativistic hadrons

as quark bound states

That is necessary for solution of a task 1.

1. Equation of bound states

2. Interquark potential

Task 1.

1.1 In the present study for description bound two-particle system, we make use of the instant form of relativistic Hamiltonian dynamics (RHD). The fundamentals of RHD are considered in detail elsewhere ***B.D.Keister and W.N.Polyzou, Adv. Nucl.Phys.***, V.20 ***(1991), P.225***
L.A. Kondratyuk, F.M.Lev and V.G.Soloviev, Few-Body Syst. V.7,(1989) P.55.
W.N.Polyzou, Ann. Phys.(N.Y.) V.193. (1989), P.369. .....

1.2 For pseudoscalar and vector mesons we use one-gluon-exchange-plus-linear-confinement potential motivated by QCD with smearing parameter ***S. Godfrey and N. Isgur Phys. Rev. D*** 32, ***(1985) P.185***.

Constituents of task:

The polarizabilities of relativistic bound system

Task 2.

Description of the quark bound systems

in external electric field

That is necessary for solution of a task 2.

1. Dipole interaction in relativistic case

2. Calculation method of the electromagnetic polarizabilities

Task 2.

2.1 The dipole interaction is constructed with help of the scattering amplitude of interaction between quarks and external electric fields.

2.2 To evaluate the static electric and magnetic polarizability we apply the procedure that includes the calculations of contributions by means of the perturbation theory and the variation method : ***V.V. Andreev and N.V.Maksimenko, Proc. of Int. School-seminar "Actual problems of particle physics". August 7-16, (2001), Vol. 2, E1, 2-2002-166, Dubna, JINR, 2002. P.128-139*** (see, also ***W. Lucha and F. F. Schöberl, ArXiv:hep-ph/0204325***).

Task 1.

The bound two-particle system in the RHD

The RHD description of mesons is the form of relativistic constituent quark model with phenomenological interaction. The relativistic Hamiltonian dynamics differs from the ordinary non-relativistic quantum mechanics, as the main requirement for the operators of the complete set of states is the one that the generators that make up the operators should follow the algebra of Poincare$'$ group. In the our approach the mesons are represented as bound states of relativistic point-like a quark q and an antiquark $\bar{Q}$, with masses m_q and m_Q and electric charges e_q and e_Q.

Equation of the bound system

The Hamiltonian $\hat{H}$ is assumed to be the sum of an relativistic kinetic energy operator $T(\mathbf{k})$ that represents the invariant mass of two noninteracting quarks plus a phenomenological interaction $\hat{V}$. The kinetic energy operator has the form

$$T(\mathbf{k}) = \sqrt{m_q^2 + \mathbf{k}^2} + \sqrt{m_Q^2 + \mathbf{k}^2}\ , \tag{9}$$

where

$$\mathbf{k} = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2) + \frac{\mathbf{P}}{M_0}\left(\frac{s - M_0\left[\omega_{m_Q}(\mathbf{p}_2) - \omega_{m_q}(\mathbf{p}_1)\right]}{\omega_{M_0}(\mathbf{P}) + M_0}\right) \tag{10}$$

is the relative momentum. In Eq.(10) the momentum $\mathbf{P}$ is the total momentum of the free-system $\mathbf{P} = \mathbf{p_1} + \mathbf{p_2}$ and $M_0^2 = \omega_{M_0}^2(\mathbf{P}) - \mathbf{P}^2$, $\omega_m(\mathbf{p}) = \sqrt{m^2 + \mathbf{p}^2}$, $s = m_Q^2 - m_q^2$.

The eigenvalue problem for the mass of a bound system Ψ with momentum $\mathbf{Q}$, spin J and spin's projection μ can be written as follows ***B.D.Keister and W.N.Polyzou, Adv. Nucl.Phys., V.20 (1991), P.225***:

$$\hat{M} \mid \Psi_{\mathbf{Q},J,\mu} > \equiv \left(T(\mathbf{k}) + \hat{V}\right) \mid \Psi_{\mathbf{Q},J,\mu} >= M_\Psi \mid \Psi_{\mathbf{Q},J,\mu} >\ . \tag{11}$$

Here M_Ψ represents the mass a particle (meson) with spin J.

Equation of the bound system

In practical applications it is convenient to introduce wave functions

$$\begin{aligned}\langle \mathbf{p}_1, \lambda_1; \mathbf{p}_2, \lambda_2 \left| \Psi_{\mathbf{Q},J,\mu} \right\rangle \equiv \Psi^{J\mu}_{\mathbf{Q};\lambda_1,\lambda_2}\left(\mathbf{p}_1, \mathbf{p}_2\right) = \\ = \delta(\mathbf{Q}-\mathbf{p}_1-\mathbf{p}_2)\tilde{\Phi}^{J\mu}_{\mathbf{Q};\lambda_1\lambda_2}\left(\mathbf{p}_1, \mathbf{p}_2\right)\end{aligned} \qquad (12)$$

are determined through the basis of direct product $|\mathbf{p}_1, \lambda_1; \mathbf{p}_2, \lambda_2\rangle$ or wave functions

$$\left\langle \mathbf{P}, \mathbf{k}, \lambda_1, \lambda_2 \right| \left. \Psi_{\mathbf{Q},J\mu} \right\rangle = \delta\left(\mathbf{P}-\mathbf{Q}\right) \Phi^{J\mu}_{\mathbf{Q};\lambda_1,\lambda_2}\left(\mathbf{k}\right), \qquad (13)$$

are determined through through the basis

$$\left|\mathbf{P}, \mathbf{k}, \lambda_1, \lambda_2\right\rangle = \sqrt{\frac{\omega_{m_q}\left(\mathbf{p}_1\right)\omega_{m_Q}\left(\mathbf{p}_2\right) M_0}{\omega_{m_q}\left(\mathbf{k}\right)\omega_{m_Q}\left(\mathbf{k}\right)\omega_{M_0}\left(\mathbf{P}\right)}} \left|\mathbf{p}_1, \lambda_1, \mathbf{p}_2, \lambda_2\right\rangle \ . \qquad (14)$$

The correlation between wave functions (12),(13) is easily obtained by means of Clebsh-Gordan coefficients of Poincare group .

In this approach a meson with the momentum $\mathbf{Q}$, mass M_ψ and spin J is decribed by the wave function (13) of quark-antiquark state, which satisfies the equation

$$\sum_{\lambda_1,\lambda_2} \int < \mathbf{k}, \sigma_1, \sigma_2 \parallel V \parallel \ \mathbf{k}', \lambda_1, \lambda_2 > \Phi^{J\mu}_{\mathbf{Q};\lambda_1\lambda_2}(\mathbf{k}')\, d\mathbf{k}' =$$

$$\left(M_\psi - \sqrt{\mathbf{k}^2 + m_q^2} - \sqrt{\mathbf{k}^2 + m_Q^2}\right) \Phi^{J\mu}_{\mathbf{Q};\sigma_1\sigma_2}(\mathbf{k}) \tag{15}$$

If the bound system is in an external electromagnetic field, the equation (11) is modified to the following form (see below):

$$\left(T(\mathbf{k}) + \hat{V} + \hat{V}_{\mathbf{em}}\right) \mid \Psi^{\mathbf{E}}_{\mathbf{P},J,\mu} >= (M_\Psi + \Delta\varepsilon) \mid \Psi^{\mathbf{E}}_{\mathbf{P},J,\mu} >, \tag{16}$$

where operator $\hat{V}_{em}$ is the dipole interaction with external electromagnetic (electric or magnetic) field , and $\Delta\varepsilon$ is the correction to the energy of the ground state of the described wave function $\Psi_{\mathbf{P},J,\mu}$.

Interquark Potential

In applications the interaction $\hat{V}$ is determined by the physics of the system being modeled. For pseudoscalar and vector mesons we use one-gluon-exchange-plus-linear-confinement potential motivated by QCD with smearing parameter: ***S. Godfrey and N. Isgur Phys. Rev. D* 32, *(1985) P.185***.

The interquark potential in the coordinate representation is the sum of Coulomb-like, confining smeared potentials and spin-spin interaction part (the contribution of tensor forces responsible for the transition between the states with different l was neglected)

$$\hat{V}_{qQ}(r) = \hat{V}_{Coulomb}(r) + \hat{V}_{linear}(r) + \hat{V}_{SS}(r). \tag{17}$$

with $r = |\mathbf{r}|$.

The smeared Coulomb part of interquark potential is determined by

$$\hat{V}_{Coulomb}(r) = -\frac{4}{3}\sum_{k=1}^{3}\frac{\alpha_k}{r}\,\mathrm{erf}(\tau_k\, r), \tag{18}$$

$$\mathrm{erf}(x) = \left(\frac{2}{\sqrt{\pi}}\right)\int_0^x \exp(-t^2)dt$$

with the error function $\mathrm{erf}(x)$.

The parameterization of the running coupling constant $\alpha_s\,(Q^2)$ in this approach have the convenient form

$$\alpha_s(Q^2) = \sum_{k=1}^{3}\alpha_k \exp(-Q^2/4\gamma_k^2), \tag{19}$$

here $\alpha_1 = 0.43209, \alpha_2 = 0.13089, \alpha_3 = 0.11657$, $\gamma_1 = 0.65141$, $\gamma_2 = 187.13991, \gamma_3^2 = 5.45638$,

$$\frac{1}{\tau_k^2} = \frac{1}{\gamma_k^2} + \frac{1}{\sigma^2},$$

and σ is the smearing parameter. The confining part of interaction has the form:

$$\hat{V}_{linear}(r) = b\, r \left[\frac{\exp(-\sigma^2 r^2)}{\sqrt{\pi}\sigma r} + \left(1 + \frac{1}{2\sigma^2 r^2}\right) \mathrm{erf}(\sigma\, r) \right] + w_0. \tag{20}$$

Spin-spin part of the interaction is determined by

$$\hat{V}_{SS}(r) = -\frac{32\sigma^3}{9\sqrt{\pi} m_q m_Q} (\mathbf{S}_q \mathbf{S}_Q) \exp(-\sigma^2 r^2) \sum_k \alpha_k \mathrm{erf}(\gamma_k r), \tag{21}$$

where S_q, S_Q are spin operators of quarks.

Task 2.

Dipole interaction

The dipole interaction is constructed with help of the scattering amplitude according to the below prescription.

1. Compute the scattering amplitude R_{fi}, which is defined in terms of the S-matrix element by the decomposition

$$S_{fi} = \delta_{fi} - \mathrm{i}2\pi\delta\left(E_f - E_i\right) R_{fi}, \tag{22}$$

where i and f denote of initial and final state, respectively.

2. The potential $\hat{V}$ can be extracted from Eq.(22) with help of relation

$$\langle f|\, \hat{V}\, |i\rangle = R_{fi}. \tag{23}$$

In impulse approximation the scattering amplitude for the dipole interaction is given by the two Feynman diagrams (see Fig.1).

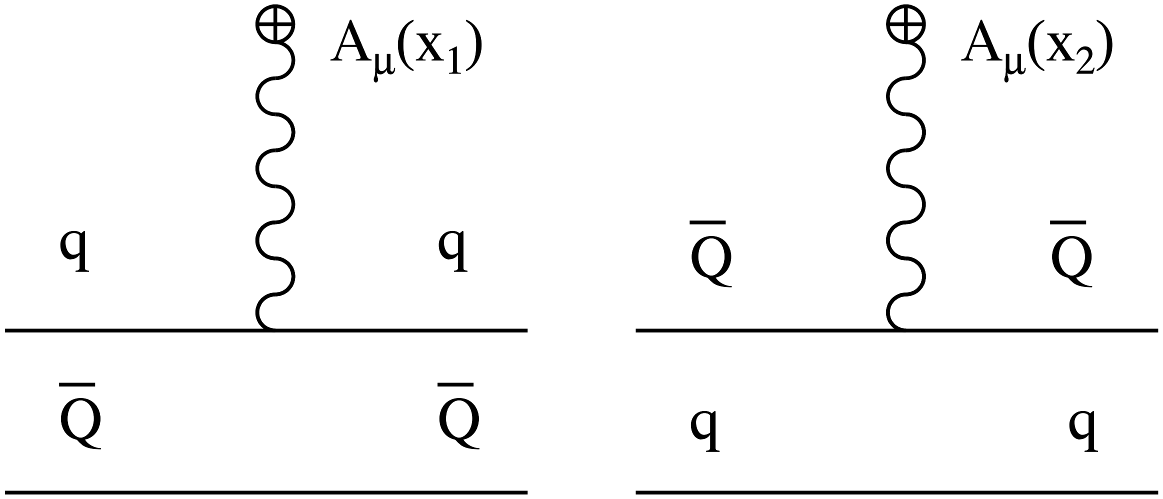

Figure 1: Feynman diagrams of dipole interaction (impulse approximation)

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If the electromagnetic field is the external static electric with strength $\mathbf{E}$, the operator $\hat{A}(x)$ is determined by

$$\hat{A}(x) = \left(A^0(x), \mathbf{0}\right) = (-\mathbf{r}\mathbf{E}, \mathbf{0}) \,. \tag{24}$$

and if the electromagnetic field is the external static magnetic with strength $\mathbf{H}$, the operator $\hat{A}(x)$ is determined by

$$\hat{A}(x) = (0, \mathbf{A}(x)) = (0, 1/2\,[\mathbf{H} \times \mathbf{r}]) \,. \tag{25}$$

The corresponding operators of dipole electromagnetic interaction have following forms

$$\begin{aligned}
&\langle \mathbf{p}_1', \lambda_1', \mathbf{p}_2', \lambda_2' | \, \hat{V}_{\mathbf{E}} \, | \mathbf{p}_1, \lambda_1, \mathbf{p}_2, \lambda_2 \rangle = \\
&\frac{\mathrm{i} e_q \delta(\mathbf{p}_2' - \mathbf{p}_2)\, \delta_{\lambda_2' \lambda_2}}{\sqrt{4\omega_{m_q}(\mathbf{p}_1)\,\omega_{m_q}(\mathbf{p}_1')}} \overline{u}_{\lambda_1'}(p_1')\, \gamma_0 u_{\lambda_1}(p_1) \left((\mathbf{E}\nabla_{\mathbf{p}_1})\, \delta(\mathbf{p}_1' - \mathbf{p}_1) \right) + \\
&+ \frac{\mathrm{i} e_Q \delta(\mathbf{p}_1' - \mathbf{p}_1)\, \delta_{\lambda_1' \lambda_1}}{\sqrt{4\omega_{m_Q}(\mathbf{p}_2)\,\omega_{m_Q}(\mathbf{p}_2')}} \overline{v}_{\lambda_2'}(p_2)\, \gamma_0 v_{\lambda_2}(p_2') \left((\mathbf{E}\nabla_{\mathbf{p}_2} \delta(\mathbf{p}_2' - \mathbf{p}_2)) \right) .
\end{aligned} \tag{26}$$

$$
\begin{aligned}
&\left\langle \mathbf{p}_1', \lambda_1', \mathbf{p}_2', \lambda_2' \right| \hat{V}_{\mathbf{H}} \left| \mathbf{p}_1, \lambda_1, \mathbf{p}_2, \lambda_2 \right\rangle = \\
&= \frac{\mathrm{i} e_q \delta\left(\mathbf{p}_2' - \mathbf{p}_2\right) \delta_{\lambda_2' \lambda_2}}{4\sqrt{\omega_{m_q}\left(\mathbf{p}_1\right) \omega_{m_q}\left(\mathbf{p}_1'\right)}} \left(\left(\left[\mathbf{j}_1 \times \mathbf{H}\right] \nabla_{\mathbf{p}_1}\right) \delta\left(\mathbf{p}_1' - \mathbf{p}_1\right)\right) + \\
&+ \frac{\mathrm{i} e_Q \delta\left(\mathbf{p}_1' - \mathbf{p}_1\right) \delta_{\lambda_1' \lambda_1}}{4\sqrt{\omega_{m_Q}\left(\mathbf{p}_2\right) \omega_{m_Q}\left(\mathbf{p}_2'\right)}} \left(\left(\left[\mathbf{j}_1 \times \mathbf{H}\right] \nabla_{\mathbf{p}_2} \delta\left(\mathbf{p}_2' - \mathbf{p}_2\right)\right)\right) \ ,
\end{aligned} \tag{27}
$$

where

$$
\mathbf{j}_1 = \bar{u}_{\lambda_2'}\left(p_2\right) \gamma \, u_{\lambda_2}\left(p_2'\right) \ , \tag{28}
$$

$$
\mathbf{j}_2 = \bar{\upsilon}_{\lambda_2'}\left(p_2\right) \gamma \, \upsilon_{\lambda_2}\left(p_2'\right) \ . \tag{29}
$$

Task 2.

Estimation of static electric polarizability

We apply the procedure that includes the analytic calculations of contributions by means of the standard variation method: ***V.V. Andreev and N.V.Maksimenko, Proc. of Int. School-seminar "Actual problems of particle physics" Vol. 2, E1, 2-2002-166, Dubna, JINR, 2002. P.128***

The final results is

$$\alpha_0 \approx \frac{2B^2/\mathbf{E}^2}{\langle\Psi_0|\left[\Delta\hat{H},\hat{H}_0\right]\Delta\hat{H}\,|\Psi_0\rangle}\ , \qquad (30)$$

where $B = \langle\Psi_0|\,\Delta\hat{H}^2\,|\Psi_0\rangle$. Here the operator $\hat{H}_0$ is Hamiltonian of unperturbed system and $\Delta\hat{H}$ is a certain small addition (the perturbation).

Static polarizabilities of the charged pseudoscalar meson

Hence the problem on calculation of bounds of an electric polarizability is divided into two parts. The first part consists in a determination of mass spectrum and wave functions of two-particle system (so called “spectroscopic part”). The next step consists in an evaluation of the polarizabilities with the help of the ground wave function obtained in the first part (“estimation part”). The spectroscopic results of the model we shall support by an analysis of π^+-meson lepton constant.

The resulting equation (16) of bound quark-antiquark system with external electric field $\mathbf{E}$ is given by the following expression in the rest system of pseudoscalar meson $(\mathbf{Q} = \mathbf{0})$

$$\sum_{\lambda_1,\lambda_2}\int < \mathbf{k},\sigma_1,\sigma_2 \parallel \hat{V}_{qQ} \parallel \ \mathbf{k}',\lambda_1,\lambda_2 > \Phi^{J=0}_{\mathbf{Q=0};\lambda_1\lambda_2}\left(\mathbf{k}',\mathbf{E}\right)d\mathbf{k}' -$$
$$-1/2\left(\mathbf{ED}\right)\Phi^{J=0}_{\mathbf{Q=0};\lambda_1,\lambda_2}\left(\mathbf{k},\mathbf{E}\right) =$$
$$= \left(M_{meson} + \Delta\varepsilon - 2\sqrt{\mathbf{k}^2 + m_q^2}\right)\Phi^{J=0}_{\mathbf{Q=0};\sigma_1\sigma_2}\left(\mathbf{k},\mathbf{E}\right) \tag{31}$$

where the potential $\hat{V}_{qQ}$ and the operator $(\mathbf{ED})$ are determined by Eqs. (17),(18),(20),(21) and

$$\Delta\hat{H}_{\mathbf{E}} = -1/2\left(\mathbf{ED}\right) \equiv -\mathrm{i}/2\; q_{12}\left(\mathbf{E}\nabla_{\mathbf{k}}\right) \tag{32}$$

with $q_{12} = e_q - e_Q$ and $M_0 = 2\sqrt{m_q^2 + \mathbf{k}^2}$.

If electromagnetic field is the static magnetic field $\mathbf{H}$, we have that

$$\Delta\hat{H}_{\mathbf{H}} \equiv -1/2\left(\mathbf{MH}\right) = \frac{\mathrm{i}q_{12}}{4\sqrt{\mathbf{k}^2 + m_q^2}}\left(\mathbf{H}\left[\mathbf{k}\times\nabla_{\mathbf{k}}\right]\right)\ . \tag{33}$$

Spectroscopic part

The discrete eigenvalues and corresponding wave functions of the operator represented by the Hamiltonian $T+\hat{V}$ are approximately determined with help of the variational method. It the case of RHD this method is reduced to the calculation of the minimum of the functional

$$M_{(N)}\left(\beta\right) \equiv \langle \Phi^{(N)}|T\left(\mathbf{k}\right)|\Phi^{(N)}\rangle + \langle \Phi^{(N)}|\hat{V}_{qQ}|\Phi^{(N)}\rangle = M_{meson}. \tag{34}$$

Here β is parameter of trial function, M_{meson} is a meson mass.

As trial functions of state with $J=0\,(l=s=0)$ we use the wave functions of harmonic oscillator

$$\Phi^{(N)}_{\lambda_1,\lambda_2}\left(\mathbf{k},\beta\right) = \delta_{\lambda_1,-\lambda_2}\frac{\lambda_1-\lambda_2}{\sqrt{2}}\phi^{(N,l=0)}\left(\mathbf{k},\beta\right), \tag{35}$$

where

$$\phi^{(N,l)}\left(\mathbf{k},\beta\right)=\sqrt{\frac{2N!}{\beta^{3}\Gamma(N+l+3/2)}}\times$$
$$\times\exp\left(-\frac{k^{2}}{2\beta^{2}}\right)\left(\frac{k}{\beta}\right)^{l}Y_{l}^{m}\left(\hat{\mathbf{k}}\right)L_{N}^{l+1/2}\left(\frac{k^{2}}{\beta^{2}}\right) \tag{36}$$

with the generalized Laguerre polynomials $L_{n}^{a}(x)$ and the spherical harmonic $Y_{l}^{m}\left(\hat{\mathbf{k}}\right)=Y_{l}^{m}\left(\theta_{k},\varphi_{k}\right)$.

We have that

$$\phi^{(0,0)}(k,\beta) = \frac{1}{\pi^{3/4}\beta^{3/2}}\exp\left(-\frac{k^{2}}{2\beta^{2}}\right), \tag{37}$$

for ground state.

Our relativistic approach to the calculation of the electric polarizability for pseudoscalar mesons contains the set of the model parameters: masses of quarks m_q and m_Q as well as parameters of the interaction operator $V_{qQ}(r)$

i.e. b , σ and w_0 (see Eqs.(17),(18),(20),(21)).

Let us discuss the choice of the values of these parameters in our calculation. The parameters of the interaction operator are determined usually in a phenomenological way, from the description of meson spectra.

The parameter of the linear part of interaction in (20) is $b = 0.18$ GeV^2 in the most current calculations . The smearing parameter σ and the parameter of the trial functions β are determined from

$$\partial M_{(0)}\left(\beta,\sigma\right)/\partial\beta\mid_{\beta_{min},\tilde{\sigma}} = 0\ , \tag{38}$$

$$M_{(0)}^{S=1}\left(\beta,\sigma\right) - M_{(0)}^{S=0}\left(\beta,\sigma\right)\mid_{\beta_{min},\tilde{\sigma}} = \Delta M_{exp}\ , \tag{39}$$

where the first equation is the minimum condition and the second equation represents a requirement,that the difference of the masses of spin-singlet ($S = 0$) and spin-triplet ($S = 1$) quarkonium states is equal to an experimental value of mesons. The parameter w_0 in Eq.(20) is determined by the condition

$$M_{(0)}\left(w_0, \beta_{min}, \tilde{\sigma}\right) = M_{exp}, \tag{40}$$

where M_{exp} is the experimental value of the pseudoscalar meson mass.

The mass of quarks can be fixed from the description of lepton decay constant in instant form RHD ***A.F. Krutov, Yad.Fiz* V.60, *N8, (1997) P.1443. (Phys.At.Nuclei*** **V.60,** ***(1997) P.1305)***;
V.V. Andreev, Vesci NAN, Ser.fiz.-mat.navuk **N 2,*(2000) P.93***

$$f_P\left(m_q, m_Q, \beta\right) = \frac{1}{\pi\sqrt{6}} \int_0^\infty dk\, k^2 \phi^{(0,0)}\left(k, \beta\right)$$
$$\sqrt{\frac{\left(m_q + m_Q\right)^2 - \left(\omega_{m_q}\left(\mathbf{k}\right) - \omega_{m_Q}\left(\mathbf{k}\right)\right)^2}{\omega_{m_q}\left(\mathbf{k}\right)\omega_{m_Q}\left(\mathbf{k}\right)\left(\omega_{m_q}\left(\mathbf{k}\right) + \omega_{m_Q}\left(\mathbf{k}\right)\right)}}\ . \tag{41}$$

The potential parameters of $\pi^{\pm}$–mesons

Let us apply this plan for the description of $\pi^{\pm}$- -mesons. From experimental requirements (see ***D.E. Groom et al., Eur.Phys.J. C* 15, *(2000) P.1***)

$$M^{\pi^{\pm}}_{exp} = 139.56995 \pm 0.00035 \text{ MeV},$$
$$\Delta M_{exp} = M^{\rho} - M^{\pi^{\pm}} = 627.33 \text{ MeV},$$
$$f_{\pi^{\pm}} = 130.7 \pm 0.1 \pm 0.36 \text{ MeV} \tag{42}$$

we receive from Eqs.(38),(40),(41) that

$$\beta^{\pi^{\pm}}_{min} = 0.4332 \text{ GeV}, \quad \tilde{\sigma} = 0.3518 \text{ GeV},$$
$$w_0 = -0.9391 \text{ GeV}, \; m_u = m_d = 0.201 \text{ GeV}. \tag{43}$$

The static electric polarizability α_0

To obtain the electric polarizability α_0 of charged π–mesons it is necessary to calculate following values

$$B_{(0)} = \left\langle \Phi^{(0)} \right| \Delta\hat{H}^2 \left| \Phi^{(0)} \right\rangle =$$

$$= 1/4 \sum_{\lambda_1,\lambda_2} \int \mathrm{d}^3\mathrm{k}\Phi^{*(0)}_{\lambda_1,\lambda_2}(\mathbf{k})\,(\mathbf{ED})\,(\mathbf{ED})\,\Phi^{(0)}_{\lambda_1,\lambda_2}(\mathbf{k}) \qquad (44)$$

$$A_{(0)} \equiv \left\langle \Phi^{(0)} \right| \left[\Delta\hat{H}, \hat{H}_0\right] \Delta\hat{H} \left| \Phi^{(0)} \right\rangle =$$

$$= 1/4 \sum_{\lambda_1,\lambda_2} \int \mathrm{d}^3\mathrm{k}\Phi^{*(0)}_{\lambda_1,\lambda_2}(\mathbf{k})\,[(\mathbf{ED})\,, T(\mathbf{k})]\,(\mathbf{ED})\,\Phi^{(0)}_{\lambda_1,\lambda_2}(\mathbf{k}) \; . \qquad (45)$$

The result is

$$B_{(0)} = -\frac{\mathbf{E}^2 \langle q_{12} \rangle^2}{12} \int_0^\infty \mathrm{d}k k^2 \phi^{*(0)}(k) \Delta \phi^{(0,0)}(k)$$

$$A_{(0)} = -\frac{\mathbf{E}^2 \langle q_{12} \rangle^2}{6} \int_0^\infty \mathrm{d}k k \phi^{*(0,0)}(k) \frac{1}{\omega_{m_q}(\mathbf{k})} \frac{\partial \phi^{(0,0)}(k)}{\partial k}, \tag{46}$$

For the bound system with equal masses of quark $m_q = m_Q = m$, which described by the wave function (35) we obtain analytic result

$$\alpha^0 \approx \frac{3 \langle q_{12} \rangle^2 \sqrt{\pi} \exp\left(-V^2/2\right)}{16 m^2 \beta \left(V^2 K_0\left(V^2\right) - \beta^2 \left(V^2 - 1\right) K_1\left(V^2\right)\right)}, \tag{47}$$

where $V = m/\beta$ and $K_n(z)$ is the Bessel function of second kind. For $\pi^\pm$ meson $(u\bar{d})$ the value $\langle q_{12}^2 \rangle$ is $\langle q_{12}^2 \rangle = e^2/9$ with electric charge e $(e^2 = 1/137.036)$.

Using the Eq.(43) we obtain that

$$\alpha_0^{\pi^\pm} \approx \mathbf{0.14 * 10^{-4}}\ \mathrm{Fm}^3 \tag{48}$$

Magnetic static polarizability β_0

Since the expectation value for spherically symmetric states

$$\left\langle \phi^{(N,0)}\left(k\right) \left| \Delta \hat{H}_{\mathbf{H}} \right| \phi^{(0,0)}\left(k\right) \right\rangle = 0 \tag{49}$$

with magnetic part of perturbation

$$\Delta \hat{H}_{\mathbf{H}} \equiv -1/2\left(\mathbf{MH}\right) = \frac{\mathrm{i}q_{12}}{4\sqrt{\mathbf{k}^2 + m_q^2}}\left(\mathbf{H}\left[\mathbf{k} \times \nabla_{\mathbf{k}}\right]\right) \tag{50}$$

the magnetic static polarizability β_0 is equal zero i.e.

$$\beta_0 = 0. \tag{51}$$

Relativistic corrections

Let us discuss in brief role of relativistic corrections in (47). We can select three basic domain depending on numerical values between parameters m and β: nonrelativistic range- $(m \gg \beta)$, ultrarelativistic range - $(m \ll \beta)$ and semirelativistic range - $(m \backsim \beta)$. As will readily be observed, the result of our calculations give a value $W = \beta/m \approx 2.2$, which relevant to relativistic behaviour.

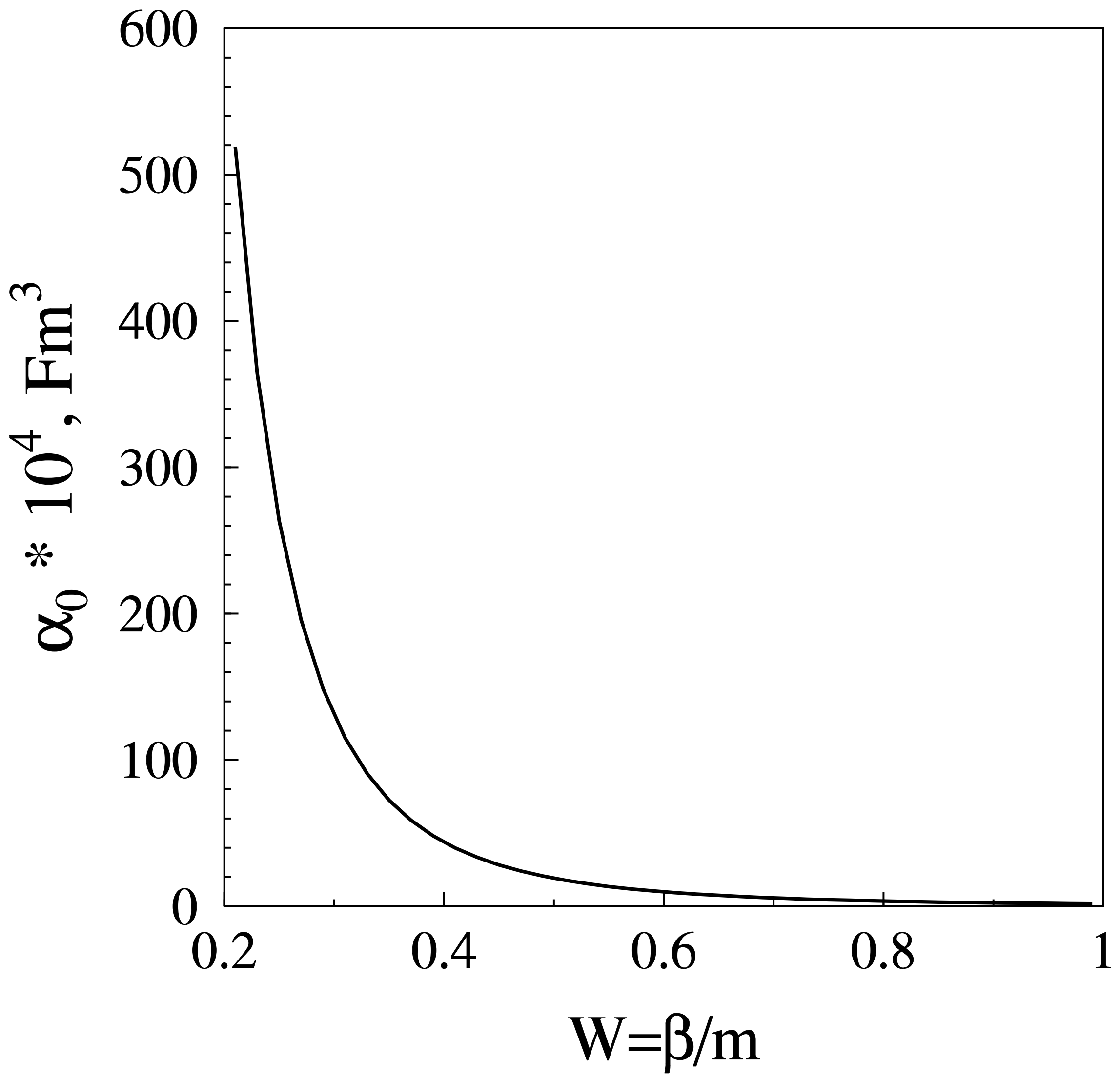

Figure 2: α_0 as function $W = \beta/m$ at fixed $m = 0.201$ GeV

Let us the results of numerical calculations of lower bounds of static polarizability . Fig.2 shows the function α_0 from (47) as function of $W = \beta/m$. We can see that if parameter W decreases (nonrelativistic range), the value of a polarizability α_0 is increases and accordingly if the value V increases (ultrarelativistic range), we have that the value of α_0 is decreases. From Fig.2 we conclude that relativistic corrections are negative.

This result is in agreement with that obtained in Ref. ***W. Lucha and F. F. Schöberl, Phys. Lett. B*** 544 ***(2002) P.380; ArXiv:hep-ph/0204325*** with the use of the relativistic kinetic energy of quarks and Ref. ***R.N.Lee, A.I. Milstein and M.Schumacher, hep-ph/0101240.*** with use of the Compton scattering on the QED bound state of two charged particles.

Compton electric and magnetic polarizabilities

Compton electric polarizability

Let us term $\Delta\alpha$ (3) in the framework of a description of mesons as bound states of point-like constituent quarks with help of instant form of RHD.

The relativistic expression for pion mean square radius $\langle r_\pi^2\rangle$ was obtained in ***A.F. Krutov and V.E. Troitsky, J.Phys.G: Nucl. Part. Phys. 19 (1993) P.L127-L131*** with wave function $\phi^{0,0}(\mathbf{k},\beta)$ (37)

$$\left\langle r_\pi^2 \right\rangle = \frac{3}{8\beta^2} H\left(V\right), \quad V = m/\beta \tag{52}$$

$$H\left(V\right) = \exp\left(V^2\right)\left(\sqrt{\pi} V \operatorname{erfc}\left(V\right) + E_1\left(V^2\right)\right). \tag{53}$$

for point-like quarks in instant form of RHD. Here

$$E_1(x) = \int_x^\infty \frac{\exp(-t)}{t} dt, \;\; x > 0,$$

$$\mathrm{efrc}(x) = 1 - \frac{2}{\sqrt{\pi}} \int_0^x \exp\left(-t^2\right) dt. \qquad (54)$$

Using the numerical values of m_u, β (43) we obtain

$$\left\langle r_\pi^2 \right\rangle = 0.153 \ \mathrm{Fm}^2. \qquad (55)$$

The value $\langle r_\pi^2 \rangle$ (54) is inconsistent with experimental electric radius of the $\pi^\pm$-meson ***S.R.Amendolia et al., Nucl.Phys. B 277 (1986) P.186***

$$\left\langle r_\pi^2 \right\rangle_{exp} = (0.44 \pm 0.01) \ \ \mathrm{Fm}^2 \qquad (56)$$

but we should take into account, that neglect sizes of quarks. For correct description of experimental data of $\langle r_\pi^2 \rangle_{exp}$ it is necessary to take into account of the quark form-factors, that reduces in additional positive term in the $\langle r_\pi^2\rangle$ (see,***A.F. Krutov, Yad.Fiz* V.60, *N8,* *(1997) P.1443.* *(Phys.At.Nuclei* V.60, *(1997) P.1305)***).

The correction term in this approach have the following numerical value

$$\Delta\alpha = 5.26 \times 10^{-4}\ \mathrm{Fm}^3. \tag{57}$$

Therefore, we obtain that the static polarizability α_0 of bound system with point-like quarks is much less than corrections term $\Delta\alpha$. Should be noted, that the model Compton polarizability $\bar{\alpha}$

$$\bar{\alpha} = \left[\underbrace{0.14}_{\alpha_0} + \underbrace{5.26}_{\Delta\alpha}\right] \times 10^{-4}\ \mathrm{Fm}^3 = 5.4 \times 10^{-4}\ \mathrm{Fm}^3 \tag{58}$$

is in agreement with average experimental value $\bar{\alpha}_{exp} = (4.3 \pm 1.2)\times 10^{-4}\ \mathrm{Fm}^3$.

Compton magnetic polarizability

Let us consider the Compton magnetic polarizability $\bar{\beta} = \beta_0 + \Delta\beta$. We obtained that the static magnetic polarizability $\beta_0 = 0$. Hence we find for the additional term

$$\Delta\beta = -\sum_i \frac{e_i^2 \langle\psi| r_i^2 \mid \psi\rangle}{6m_i} - \frac{\langle\psi \mid \mathbf{D}^2 \mid \psi\rangle}{2M_P} \tag{59}$$

following expression

$$\begin{aligned}\Delta\beta = &-\frac{\left\langle e_q^2 + e_Q^2 \right\rangle}{16V\beta^3} - \frac{3\left\langle q_{12}^2 \right\rangle}{4M_\pi \beta^2} - \\ &-\frac{\left\langle e_q^2 + e_Q^2 \right\rangle}{24V\beta^3} \left(-1 + 2V^2 - 2\sqrt{\pi}V^3 \exp\left(V^2\right)\left(-1 + \operatorname{erf}\left(V\right)\right)\right)\end{aligned} \tag{60}$$

The correction term in our approach have the following numerical value

$$\Delta\beta = -2.03 \times 10^{-4}\ \mathrm{Fm}^3. \tag{61}$$

There are many the theoretical predictions for the magnetic polarizability

$$\bar{\beta}^{\pi^{\pm}} = \begin{cases} -3.41, \textit{\textbf{M.A.Ivanov,T.Mizutani Ph.Rev.D45,1580,(1992)}} \\ -3.0 \div -7.3, \textit{\textbf{M.K.Volkov,A.E.Radzhabov,hep-ph 0403313}} \\ -2.1, \textit{\textbf{U.Burgi,Nucl.Phys. B479 (1996)}} \end{cases} \tag{62}$$

in units of 10^{-4} Fm^3. Our result $\left(\bar{\beta}^{\pi^{\pm}} = -2.03 \times 10^{-4}\ \mathrm{Fm}^3\right)$ is approximately agreement with the these calculations.

Summary

We investigate Compton electric polarizability of $\pi^{\pm}$ within the framework relativistic quark model based on description of mesons as bound states of point-like quarks with help of instant form of relativistic Hamiltonian dynamics. From this investigation we are to conclude that static electric polarizability constitutes only small fraction of total polarizability $\pi^{\pm}$-meson and that relativistic corrections are negative (see Fig.2).

We should be noted, that the obtained results are model dependent and thus the numerical values depend of potential parameters.

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