THE POLARIZATION EFFECTS IN NEUTRINO-LEPTON PROCESSES IN A MAGNETIC FIELD

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1. Introduction

1. Unlike an electrical field the magnetic field does not do work on a particle. Because the Lorentz force is always perpendicular trajectories of particles. For this reason the vacuum is stable concerning a magnetic field, i. e. even at

$$H \geq H_0 = \frac{m_e^2 c^3}{e\hbar} = 4.41 \times 10^{13} G \tag{1}$$

the magnetic field does not lead to appearance of pairs from the vacuum. Here H_0 is the characteristic Schwinger field strength, m_e is the electron mass, c is the speed of light in the vacuum, e is the elementary charge, $\hbar$ is the Planck constant. From this point of view the test of the various theories represents the interest in case of extreme magnetic fields.

2. The interest to the study of neutrino (antineutrino)-electron scattering in a strong magnetic field is connected with its important role in astrophysics. The strength of magnetic fields of neutron stars can be as large as $H \geq H_0$. Fields of $H \sim 10^{15} - 10^{17} G$ are generated in supernova explosions [ ].Therefore, for astrophysical applications, it is of great interest to take into account the effect of strong external magnetic field on neutrino (antineutrino)-electron scattering.

3. The strength of the fields achieved in laser radiation $(H \sim 10^7 - 10^8 G)$ less than the critical field H_0. Despite of it, if the particles of high energy participate in these processes, in the system of readout connected to the particle, the magnetic

field is increased in ε/mc^2 in comparison with the field in the laboratory system. In this case the characteristic field parameter

$$\chi = \frac{H}{H_0}\frac{\varepsilon}{m_e} \qquad (2)$$

becomes $\chi \geq 1$.

4. Even in laboratory experiments with beams of high-energy particles traversing single crystals, it is necessary to take into account strong internal electrical fields $\left(E \leq 10^{-4} H_0\right)$[ ]. In the system of readout moving at the velocity $\vec{v}$ a magnetic field of intensity,

$$\vec{H} = \frac{\left[\vec{E}\vec{v}\right]}{\left(1-v^2\right)^{1/2}} \qquad (3)$$

arises.

Due to the multiplier $\left(1-v^2\right)^{-1/2} >> 1$ the effective magnetic field becomes very strong and in this case the allowance for the influence of an external field is necessary. This opens the possibility for performing a laboratory test of the theory under extreme conditions.

5. Polarization effect, various quantum and non- linear effects arise in a magnetic field.

In this work we consider the neutrino-electron scattering processes that proceeds only via charged currents (W boson exchange) in the four fermion approximation of the standard Weinberg-Salam model.

2. Inverse charged lepton decay and the process $\bar{\nu}_e e \to l \bar{\nu}_l$

In the four-fermion approximation of the standard Weinberg-Salam model the amplitude of the inverse charged lepton decay in a constant external magnetic field has the form

$$M_{fi} = \frac{4}{\sqrt{2}} G_F L^{-3} J_\alpha(q)\left[\bar{u}(k')\gamma_L^\alpha u(k)\right] \qquad (4)$$

where $u(k)$ is the neutrino bispinor and

$$J^{(\alpha)}(q) = \int d^4 x e^{-iqx} \overline{\psi}_l(x) \gamma_L^\alpha \psi_e(x), \tag{5}$$

is the charged lepton current. Here ψ_l is the wave function of the charged lepton $l = e, \mu, \tau$ that is an exact solution to the Dirac equation in a constant magnetic field, $k = (\omega, k)$ and $k' = (\omega', k\ )$ are the 4-momenta of the initial and final neutrinos, respectively, which are assumed to be massles particles, the 4-momentum transfer is $q = k - k'$. $\gamma_L^\alpha = \gamma^\alpha (1 + \gamma^5)/2$ are the left-handed components of the Dirac matrices, $\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$, L is the normalization length.

With all allowance for (4) and (5) the probability of the process is given by

$$w = \frac{G_F^2}{\pi^2 L^3 \omega} \sum_f \int \frac{d^3 k'}{\omega'} \delta(\varepsilon + q_0 - \varepsilon') \{ 2(kj^*)(kj) -$$

$$- 2 \,\mathrm{Re}[(kj^*)(qj)] + \frac{1}{2} q^2 (j^* j) + i\varepsilon^{\alpha\beta\mu\nu} j_\alpha j_\beta^* q_\mu k_\nu \} \tag{6}$$

Here the current $j^\alpha(\mathbf{q})$ is connected with the current (5) by the relation

$$J^\alpha(q) = 2\pi\delta(\varepsilon + q_0 - \varepsilon') j^\alpha(\mathbf{q}). \tag{7}$$

This current is expressed in terms of the left-handed components of Dirac bispinors $\psi_l^L = \frac{1}{2}(1 + \gamma^5)\psi_l$:

$$j^\alpha(\mathbf{q}) = \int d^3 e^{i\mathbf{qr}} j^\alpha(\mathbf{r}),$$

$$j^\alpha(\mathbf{r}) = \overline{\psi}_l^L(\mathbf{r}) \gamma^\alpha \psi_e^L(\mathbf{r}). \tag{8}$$

In the standard presentation of the matrices γ^α the bispinor ψ^L is expressed with the spinor φ:

$$\psi^L = \begin{bmatrix} \varphi \\ -\varphi \end{bmatrix}, \qquad \varphi = \frac{1}{2} \begin{bmatrix} \psi_1 - \psi_3 \\ \psi_2 - \psi_4 \end{bmatrix}, \tag{9}$$

Where ψ_i are the components of the bispinor ψ.

We use the pseudo-Euclidean metric with signature $(+---)$ and the system of units where $\hbar = c = 1$. Summation in (6) is performed over the set of four charged lepton quantum numbers $f' = (n', p'_z, p'_y, \zeta')$. In the analogous set featuring unprimed quantities and referring to the electron, n is the principal quantum number $(n = 0,1,2,.......), -\infty < p_z < \infty$ is the projection of the momentum onto the magnetic-field direction chosen for the z axis, and $-\infty < p_y < \infty$ is the eigenvalue of the kinetic-momentum operator $-i\partial/\partial_y$. In the gauge that we adopted for the vector potential of the external field,

$$A^{\mu} = (0,0,xH,0) \tag{10}$$

this kinetic-momentum operator appears to be an integral of motion. The quantum number $\zeta = +1(-1)$ specifies states with definite spin orientations.

$\varepsilon = \left(m_e^2 + 2eHn + p_z^2\right)^{1/2}$, is the electron energy, $\varepsilon = \left(m_l^2 + 2eHn' + p_z'^2\right)^{1/2}$ is the charged lepton energy and $-e < 0$ is the electron charge.

Let the initial neutrino travel along the magnetic field, i.e.

$$k^{\mu} = \omega(1,0,0,1). \tag{11}$$

The choice of this kinematics simplifies the calculations considerably.

We obtain the reaction cross section in the form

$$\sigma = \frac{G_F^2}{8\pi^2\omega}\sum_{n',\zeta'}\int\frac{d^3k'}{\omega'},\delta(\varepsilon + q_0 - \varepsilon')R\ ; \tag{12}$$

where

$$R = \omega^2(F_0 - F_3)^2 - \omega(F_0 - F_3)(q_0F_0 - q_\perp F_1 - q_zF_3) +$$

$$+\frac{1}{4}q^2\left(F_0^2 - F_1^2 - F_2^2 - F_3^2\right) + \omega F_2\left[-q_\perp(F_0 - F_3) + (q_0 - q_z)F_1\right].$$

The functions F_α are expressed in terms of the Laguerre functions $I_{nn'}(x)$ as

$$\left.\begin{aligned} F_0 &= B'BI_{nn'} + A'AI_{n-1,n'-1}, \\ F_3 &= B'BI_{nn'} - A'AI_{n-1,n'-1}, \end{aligned}\right\}$$
$$\left.\begin{aligned} F_1 &= A'BI_{n,n'-1} + B'AI_{n-1,n'}, \\ F_2 &= A'BI_{n,n'-1} - B'AI_{n-1,n'}. \end{aligned}\right\} \tag{13}$$

the argument being

$$x = \frac{q_\perp^2}{2eH}, \tag{14}$$

where $q_\perp^2 = q_x^2 + q_y^2 = k_x'^2 + k_y'^2$ by virtue of (11). Expressions (13) also involve the electron spin coefficients A and B and the charged lepton spin coefficients A' and B' that are determined by the polarization states of the particles.

Below, we restrict our analysis to the case in which the electron possesses a high transverse momentum $p_\perp = (2eHn)^{1/2} >> m_l$ in the magnetic field of strength $H << H_l = \frac{m_l^2}{e}$; in this case, the motion of the electron is semiclassical because $n >> 1$. Performing a Lorentz boost along the z axis - this does not change the magnetic field **H** - and employing the kinematical condition (11), we go over to a reference frame where the longitudinal electron momentum is zero. We assume that the incident-neutrino energy ω lies in the range

$$\frac{eH}{p_\perp} << \omega << m_l \tag{15}$$

Under the above conditions, the total probability of the process depends on only two invariant parameters

$$\chi = \frac{H}{H_l}\frac{p_\perp}{m_l} = \frac{e}{m_l^3}\left[-\left(F_{\alpha\beta}p^\beta\right)^2\right]^{1/2}, \tag{16}$$

$$\kappa = \frac{2\omega\varepsilon}{m_l^2} = \frac{2kp}{m_l^2}, \tag{17}$$

where $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ is the tensor of the external field.

Conditions (11) and (15) and the inequality $H << H_l$ admit the following invariant representations

$$k^\alpha F_{\alpha\beta} = 0, \qquad \chi >> f, \qquad \kappa >> f,$$
$$f = \frac{e}{m_l^2}\left[\frac{1}{2}\left|F_{\alpha\beta}F^{\alpha\beta}\right|\right]^{1/2} << 1. \tag{18}$$

In the case being considered, the Laguerre functions $I_{nn'}$ appearing in (13) approximated by the first terms of their semiclassical asymptotic expansions in the relativistic parameter $\gamma^{-1} = \frac{m_l}{\varepsilon}$. In this approximation, we have

$$I = I_{nn'}(x) \approx \frac{\gamma^{-1}}{\sqrt{\pi}}\left(\frac{2\chi}{u}\right)^{1/3}(1+u)^{1/2}\Phi(y),$$
$$I' = I'_{nn'}(x) \approx \frac{\gamma^{-2}}{\sqrt{\pi}}\left(\frac{2\chi}{u}\right)^{2/3}\frac{(1+u)^{3/2}}{u}\Phi'(y), \tag{19}$$

where

$$\Phi(y) = \frac{1}{2\sqrt{\pi}}\int_{-\infty}^{\infty} dt \exp\left[i\left(yt + \frac{t^3}{3}\right)\right] \tag{20}$$

is the Airy function of the argument

$$y = \left(\frac{u}{2\chi}\right)^{2/3}\left(1 + \tau^2 + \frac{\kappa_0 - \kappa}{u}\right). \tag{21}$$

We also have

$$\kappa_0 = 1 - \left(\frac{m_e}{m_\mu}\right)^2, \; \kappa_0 = 1 - \left(\frac{m_e}{m_l}\right)^2, \; \Phi'(y) = \frac{d\Phi(y)}{dy}. \tag{22}$$

In the above expressions, we used the invariant spectral variable

$$u=\frac{\chi}{\chi'}-1=\frac{p_\perp}{p'_\perp}-1\approx\frac{\omega'}{\varepsilon-\omega'} \qquad (23)$$

and the invariant angular variable

$$\tau=\frac{e}{m_l^4}\frac{p^\alpha\widetilde{F}_{\alpha\beta}k'^\beta}{\chi-\chi'}=\gamma\frac{k'_z}{\omega'}\approx\gamma\vartheta \qquad (24)$$

where $\widetilde{F}_{\alpha\beta}=1/2\varepsilon_{\alpha\beta\lambda\sigma}F^{\lambda\sigma}$, $\vartheta=\pi/2-\theta$ and θ is the angle between **k′** и **H** (in the region that is important for our analysis, we have $|\vartheta|\leq\gamma^{-1}\ll 1$).

By using recursions relations for Laguerre functions, we find that the relations

$$I_{n,n'-1}\approx I-uI',$$

$$I_{n-1,n'}\approx I+vI', \qquad (25)$$

$$I_{n-1,n'-1}\approx I-uvI',$$

where $v=\frac{u}{1+u}$ are correct to terms of order $\sim\gamma^{-2}$.

With allowance for particle polarizations, the cross sections are given by

$$\sigma_t(\zeta',\zeta)=\frac{G_F^2}{\pi^2}m_l^2\int_0^\infty\frac{udu}{(1+u)^2}\int_{-\infty}^\infty d\tau\Big[(2\widetilde{\chi})^{-1/3}\left(1+\tau^2+2\zeta'\tau\right)\Phi^2(y)+$$

$$+(2\widetilde{\chi})^{1/3}\Phi'^2(y)+2(\tau+\zeta')\Phi(y)\Phi'(y)\Big],$$

(26)

$$\sigma_l(\zeta',\zeta)=4\frac{G_F^2}{\pi^2}m_l^2\int_0^\infty\frac{udu}{(1+u)^2}\int_{-\infty}^\infty d\tau\Big[(2\widetilde{\chi})^{-1/3}\zeta_-\left(\zeta'_+ +\zeta'_-\tau^2\right)\Phi^2(y)+$$

$$+(2\widetilde{\chi})^{1/3}\zeta_-\zeta'_-\Phi'^2(y)+2\zeta_-\zeta'_-\tau\Phi(y)\Phi'(y)\Big],$$

where $\tilde{\chi} = \frac{\chi}{u}$, $\zeta_\pm = \frac{1 \pm \zeta}{2}$, $\zeta'_\pm = \frac{1 \pm \zeta'}{2}$, the argument of the Airy functions is specified in (21, 22), and the subscripts "t" and "l" label the cross sections referring to the cases of transverse and longitudinal polarizations, respectively.

From (26), it follows that $\sigma_l \sim 1 - \zeta$ – that is, in the leading order in γ^{-1}, the process being considered is forbidden for electrons with right-hand circular polarization (ζ=1). That terms that are odd in the angular variable τ appear in the differential cross section $\frac{d^2\sigma_p}{du\,d\tau}(p = t, l)$ determined by the integrands in (26) is due to P nonconservation in weak interactions. This results in the asymmetry of final neutrino emission with respect to the *xy* plane chosen to be orthogonal to the magnetic field direction.

The inverse charged lepton decay $\nu_l e \to l \nu_e$ is related to the process $\bar{\nu}_e e \to l \bar{\nu}_l$ via crossing symmetry. The latter process is also of great interest. Its cross section $\bar{\sigma}$ is given by

$$\bar{\sigma}_t(\zeta', \zeta) = \frac{G_F^2}{\pi^2} m_l^2 \int_0^\infty \frac{u\,du}{(1+u)^4} \int_{-\infty}^{\infty} d\tau [(2\tilde{\chi})^{-1/3} \left(\delta^2 + \right.$$

$$\left. + \tau^2 + 2\zeta\tau\delta\right)\Phi^2(y) + (2\tilde{\chi})^{1/3}\Phi'^2(y) - 2(\tau + \zeta\delta)\Phi(y)\Phi'(y)\Big], \qquad (27)$$

$$\bar{\sigma}_l(\zeta', \zeta) = 4\frac{G_F^2}{\pi^2} m_l^2 \int_0^\infty \frac{u\,du}{(1+u)^4} \int_{-\infty}^{\infty} d\tau [(2\tilde{\chi})^{-1/3} \zeta'_-\left(\zeta_+\delta^2 + \right.$$

$$\left. + \zeta_-\tau^2\right)\Phi^2(y) + (2\tilde{\chi})^{1/3}\zeta'_-\zeta_-\Phi'^2(y) - 2\zeta'_-\zeta_-\tau\Phi(y)\Phi'(y)\Big],$$

where $\delta = m_e / m_l$. From (27) it follows that $\bar{\sigma}_l \sim 1 - \zeta'$; that is the process results in the production of only charged leptons having left-hand circular polarization. That the cross section σ and $\bar{\sigma}$ involve the variables ζ and ζ' in an asymmetric way is explained not only by P and C nonconservation in weak interactions but also by special kinematical features: left-handed electron neutrinos and right-handed charged lepton antineutrinos are emitted at small angles (not greater than $\left(\leq \gamma^{-1}\right)$) with respect to the ultrarelativistic-electron momentum.

Let us perform integration in (26) and (27) with respect to the angular variable τ.

For the cross section of the inverse charged lepton decay, this yields

$$\sigma_t(\zeta',\zeta)=\frac{G_F^2}{\pi^{3/2}}m_l^2\int_0^\infty \frac{udu}{(1+u)^2}\left[\frac{t}{2}\Phi_1(z)-\tilde{\chi}^{1/3}\zeta'\Phi(z)-\tilde{\chi}^{2/3}\Phi'(z)\right], \qquad (28)$$

$$\sigma_l(\zeta',\zeta)=2\frac{G_F^2}{\pi^{3/2}}m_l^2\int_0^\infty \frac{udu}{(1+u)^2}\left[\zeta_-[\zeta'_+ +\zeta'_-(t-1)]\Phi_1(z)-2\tilde{\chi}^{2/3}\zeta_-\zeta'_-\Phi'(z)\right],$$

where the argument of the Airy functions $\Phi(z)$, $\Phi'(z)=\frac{d\Phi(z)}{dz}$ and $\Phi_1(z)=\int_z^\infty dx\Phi(x)$ is

$$z=\tilde{\chi}^{-2/3}(1-t), \qquad t=\frac{\kappa-\kappa_0}{u} \qquad (29)$$

For the cross section of the process $\bar{\nu}_e e\rightarrow l\bar{\nu}_l$, we similarly obtain

$$\bar{\sigma}_t(\zeta',\zeta)=\frac{G_F^2}{\pi^{3/2}}m_l^2\int_0^\infty \frac{udu}{(1+u)^4}\left[\frac{\delta^2+t-1}{2}\Phi_1(z)++\delta\tilde{\chi}^{1/3}\zeta\Phi(z)-\tilde{\chi}^{2/3}\Phi'(z)\right],$$

(30)

$$\bar{\sigma}_l(\zeta',\zeta)=2\frac{G_F^2}{\pi^{3/2}}m_l^2\int_0^\infty \frac{udu}{(1+u)^4}\left[\zeta'_-[\zeta_+\delta^2+\zeta_-(t-1)\ ]\Phi_1(z)-\right.$$

$$\left.-2\tilde{\chi}^{2/3}\zeta'_-\zeta_-\Phi'(z)\ \right].$$

The integrands in (28) and (30) represents the spectral distribution $\frac{d\sigma_p}{du}$ and $\frac{d\bar{\sigma}_p}{du}$, respectively.

It follows from (29) that the effect of an external field on the processes $\nu_l e \to l\nu_e$ and $\bar{\nu}_e e \to l\bar{\nu}_l$ which may proceed in zero field as well, is determined by the parameter

$$\eta = \frac{\chi}{|\kappa - \kappa_0|}. \tag{31}$$

Let us consider the asymptotic behavior of the total cross sections for the processes $\nu_l e \to l\nu_e$ and $\bar{\nu}_e e \to l\bar{\nu}_l$.

For $\chi \ll 1$ and $\kappa > \kappa_0$ for the process $\nu_l e \to l\nu_e$, we have

$$\sigma_t(\zeta', \zeta) = \frac{G_F^2}{\pi} m_l^2 \left\{ \frac{(x-1)^2}{2x} + \frac{\chi^2}{x^4}\left[1 - \frac{2}{3}x(x-1)\right] - \zeta'\chi\frac{x-1}{x^2} \right\},$$

$$\sigma_l(\zeta', \zeta) = \frac{G_F^2}{\pi} m_l^2 \cdot 2\zeta_- \left\{ \zeta'_+\left(\ln x - \frac{x-1}{x}\right) + \zeta'_-(x - 1 - \ln x) + \right.$$

$$\left. + \frac{2}{3}\frac{\chi^2}{x^4}\left[\zeta'_+(3 - 2x) + 2\zeta'_- x(2 - x)\right] \right\}. \tag{32}$$

For the process $\bar{\nu}_e e \to l\bar{\nu}_l$, the asymptotic expressions are

$$\bar{\sigma}_t(\zeta', \zeta) = \frac{G_F^2}{\pi} m_l^2 \left\{ \frac{(x-1)^2}{12x^3}\left[x(1+2x) + \delta^2(2+x)\right] + \right.$$

$$\left. + \frac{\chi^2}{3x^6}\left[2x(4-3x) + \delta^2(10 - 12x + 3x^2)\right] + \zeta\chi\delta\frac{x-1}{x^4} \right\},$$

$$\bar{\sigma}_l(\zeta', \zeta) = \frac{G_F^2}{\pi} m_l^2 \cdot 2\zeta'_- \left\{ \frac{(x-1)^2}{6x^3}\left[\zeta_+\delta^2(2+x) + \zeta_- x(1+2x)\right] + \right. \tag{33}$$

$$+\frac{2\chi^2}{3x^6}\left[\zeta_+\delta^2\left(10-12x+3x^2\right)+2\zeta_- x(4-3x)\right]\Bigg\},$$

In these expressions $x = u_0 + 1 = \frac{(k+p)^2}{m_l^2}$ is the normalized Mandelstam variable. The effect of an external field is more pronounced for transversely polarized particles (contributions of order χ) than for longitudinally polarized particles (contributions of order χ^2). The inverse charged lepton process leads predominantly to the production of charged leptons whose spins are antiparallel to the direction of the field **H** ($\zeta' = -1$). A different effect is observed in the process $\bar{\nu}_e e \to l \bar{\nu}_l$: only final charged leptons having left-hand circular polarization are produced, as was indicated above see (27)].

For $\eta \ll 1$ and $\kappa < \kappa_0$ the asymptotic expressions for the cross sections are

$$\sigma_t(\zeta',\zeta)=\frac{G_F^2}{\pi\sqrt{3}}m_l^2\chi\frac{u_1}{(1+u_1)^2}\left(1-\frac{\sqrt{3}}{2}\zeta'\right)\exp\left(-2\sqrt{3}\frac{u_1}{\chi}\right),$$

$$\sigma_l(\zeta',\zeta)=\frac{G_F^2}{\pi\sqrt{3}}m_l^2\chi\frac{u_1}{(1+u_1)^2}\zeta_-\left(\zeta'_+ + 3\zeta'_-\right)\exp\left(-2\sqrt{3}\frac{u_1}{\chi}\right), \tag{34}$$

$$\bar{\sigma}_t(\zeta',\zeta)=\frac{G_F^2}{\pi\sqrt{3}}m_l^2\chi\frac{u_1}{(1+u_1)^4}\left(\frac{3+\delta^2}{4}+\frac{\sqrt{3}}{2}\delta\zeta\right)\exp\left(-2\sqrt{3}\frac{u_1}{\chi}\right), \tag{35}$$

$$\bar{\sigma}_l(\zeta',\zeta)=\frac{G_F^2}{\pi\sqrt{3}}m_l^2\chi\frac{u_1}{(1+u_1)^4}\zeta'_-\left(\zeta_+\delta^2+3\zeta_-\right)\exp\left(-2\sqrt{3}\frac{u_1}{\chi}\right).$$

For $\eta \gg 1$ and $\chi \gg 1$ the cross sections of the reactions are given by

$$\sigma_t(\zeta',\zeta)=\frac{\Gamma(2/3)}{9\pi}G_F^2 m_l^2\left[(3\chi)^{2/3}-\frac{2\Gamma(1/3)}{\Gamma(2/3)}\zeta'(3\chi)^{1/3}\right],$$

$$\sigma_l(\zeta',\zeta) = \frac{4\Gamma(2/3)}{9\pi} G_F^2 m_l^2 (3\chi)^{2/3} \zeta'_- \zeta_-, \tag{36}$$

$$\bar{\sigma}_t(\zeta',\zeta) = \frac{5\Gamma(2/3)}{81\pi} G_F^2 m_l^2 \left[(3\chi)^{2/3} + \frac{4\Gamma(1/3)}{5\Gamma(2/3)} \zeta\delta(3\chi)^{1/3} \right],$$

$$\bar{\sigma}_l(\zeta',\zeta) = \frac{20\Gamma(2/3)}{81\pi} G_F^2 m_l^2 (3\chi)^{2/3} \zeta'_- \zeta_-. \tag{37}$$

Under this conditions, the processes in question involve only electrons and final charged leptons having left-hand cicular polarizations. This corresponds to the massless limit: it follows from (16) and (17) that the factor $m_l^2 \chi^{2/3}$ is independent of m_l.

3. Annihilation of a neutrino and an antineutrino into a charged lepton and a positron in a magnetic field

Here we consider the process

$$\nu_l + \bar{\nu}_e \to l^- + e^+ \tag{38}$$

in a constant magnetic field.We study characteristic polarization effects associated with the direction specified by the external field and with the weak-current structure. Various processes induced by the inelastic scattering of ultrahigh-energy cosmic (anti) neutrinos on low-energy relic (anti) neutrinos in the Milky Way Galaxy are considered as possible sources of high-energy cosmic rays.

By using the four-fermion approximation of the Weinberg-Salam Standard Model and the Fierz identity, the amplitude of the process can be represented in the form

$$S_{fi} = \frac{4G_F}{\sqrt{2}} \frac{\bar{\upsilon}(k')\gamma_L^\alpha u(k)}{2L^3(\omega\omega')^{1/2}} J_\alpha(q). \tag{39}$$

where G_F is the Fermi constant; $u(k)$ and $v(k')$ are the bispinors of the massless neutrinos ν_l and antineutrinos $\bar{\nu}_e$ with 4-momenta $k=(\omega,k)$ and $k'=(\omega',k')$, respectively. The charged-lepton current is given by

$$J^{\alpha}(q)=\int d^4xe^{-iq\cdot x}\,\overline{\psi_{n'}^{(+)}}(x)\gamma_L^{\alpha}\psi_n^{(-)}(x)=2\pi\delta(\varepsilon'+\varepsilon-E)j^{\alpha}(\mathbf{q}), \qquad (40)$$

where the charged lepton wave functions $\psi_{n'}^{(+)}$ and the positron (negative-frequency electron) wave functions are exact solutions to the Dirac equation in a constant magnetic field. We represent the cross section for the process in the general form

$$\sigma=\frac{8\pi}{L^3}\cdot\frac{G_F^2}{k'\cdot k}\sum_{f',f}\delta(\varepsilon'+\varepsilon-E)\Big[(k'j^*)(kj)+(k'j)(kj^*)- \qquad (41)$$

$$-(k'k)(j^*j)-i\varepsilon^{\alpha\beta\mu\nu}j_{\alpha}^*j_{\beta}k'_{\mu}k_{\nu}\Big]$$

where summation is performed over the sets of four quantum numbers of the charged lepton $f'=(n',p'_z,s',\zeta')$ and four quantum numbers of the positron $f=(n,p_z,s,\zeta)$. The axisymmetric gauge of the 4-potential of the magnetic field is

$$A^{\mu}=\left(0,-\frac{1}{2}yH,\frac{1}{2}xH,0\right). \qquad (42)$$

We restrict ourselves to the case where a neutrino and an antineutrino approach each other from opposite directions in the plane orthogonal to the field **H**. Since the problem in an external field possesses axial symmetry, the choice of the x axis along the collision axis imposes no constraints on the generality of our consideration. Accordingly, the 4-momenta in the neutrino pair are then taken to be

$$k=\omega(1,1,0,0),\quad k'=\omega'(1,-1,0,0). \qquad (43)$$

in which case $q_y=q_z=0$ and the angle φ is

$$\varphi = \begin{cases} 0, & \omega > \omega' \\ \pi, & \omega' > \omega \end{cases}. \tag{44}$$

We express the cross section in terms of the functions F and F' as

$$\sigma^{(\pm)} = \frac{8}{\pi} G_F^2 eH \sum_{\zeta,\zeta'} \sum_{n,n'} \int dp_z \delta(\varepsilon + \varepsilon' - E)(F' \pm F)^2 \tag{45}$$

where the upper and the lower sign in the superscript refer to the cases of $q_x = \omega - \omega' > 0$ and $q_x = \omega - \omega' < 0$, respectively and

$$F = l_1' l_2 I_{n,n'-1} - l_2' l_1 I_{n-1,n'},$$

$$F' = l_2' l_2 I_{nn'} - l_1' l_1 I_{n-1,n'-1}. \tag{46}$$

The quantities l_k ($k = 1,2$) are expressed in terms of the spin coefficients C_i ($i = 1,2,3,4$):

$$l_1 = \frac{1}{2}(C_1 - C_3), \ \ l_2 = \frac{1}{2}(C_2 - C_4). \tag{47}$$

The case of a collision between a high-energy neutrino (antineutrino) and a low-energy antineutrino (neutrino) is of interest for astrophysical applications. Suppose that the energy of the neutrino and antineutrino, and the momentum transfer $q_\perp = |\omega - \omega'| \approx E >> m_l$ are both much greater than m_l and that the magnetic-field strength satisfies the condition $H << H_l = m_l^2 / e$. The main contribution to the total cross section for this process comes from the final-lepton states having large quantum numbers n, n'>>1 (high Landau levels).

We can introduce the field, the kinematical, and the mass parameter (κ, λ and δ, respectively)

$$\kappa = \frac{e}{m_l^3}\left[-\left(F_{\alpha\beta} q^\beta\right)^2\right]^{1/2} = \frac{q_\perp}{m_l}\frac{H}{H_l} \approx \frac{eHE}{m_l^3},$$

$$\lambda = \frac{q^2}{m_l^2} = \frac{4\omega\omega'}{m_l^2}, \quad \delta = \frac{m_e}{m_l} \tag{48}$$

and the spectral and the angular variable (υ and τ)

$$\upsilon = \frac{\chi}{\chi + \chi'} \approx \frac{\varepsilon}{\varepsilon + \varepsilon'}, \tag{49}$$

$$\tau = \frac{eq^{\alpha}\widetilde{F}_{\alpha\beta}p^{\beta}}{m_l^4(\chi + \chi')} \approx \frac{p_z}{m_l} \tag{50}$$

where

$$\chi = \frac{e}{m_l^3}\left[-\left(F_{\alpha\beta}p^{\beta}\right)^2\right]^{1/2} \approx \frac{eH\varepsilon}{m_l^3}, \quad \chi' = \chi(p \to p'), \tag{51}$$

$F_{\alpha\beta} = \partial_{\alpha}A_{\beta} - \partial_{\beta}A_{\alpha}$ is the strength tensor of an external magnetic field, and $\widetilde{F}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\lambda\sigma}F^{\lambda\sigma}$ is its dual counterpart.We note that $\upsilon \in (0,1)$ and that, in the ultrarelativistic approximation adopted here $(\gamma = E/m_l >> 1)$ $\tau \in (-\infty, \infty)$.

The cross section for the production of transversely polarized leptons are given by

$$\sigma^{(+)}(\zeta', \zeta) = \frac{G_F^2}{\pi^2}m_l^2\int_0^1 d\upsilon\frac{\overline{\upsilon}}{\upsilon}\int_{-\infty}^{+\infty}d\tau\Big[(2\overline{\kappa})^{-1/3}\tau^2\Phi^2(y) +$$

$$+ (2\overline{\kappa})^{1/3}\Phi'^2(y) - 2\tau\Phi(y)\Phi'(y)\Big], \ (\omega >> \omega').$$

$$\tag{52}$$

$$\sigma^{(-)}(\zeta', \zeta) = \frac{G_F^2}{\pi^2}m_l^2\int_0^1 d\upsilon\frac{\upsilon}{\overline{\upsilon}}\int_{-\infty}^{+\infty}d\tau\Big[(2\overline{\kappa})^{-1/3}\left(1 + \tau^2 + 2\zeta'\tau\right)\Phi^2(y) +$$

$$+ (2\overline{\kappa})^{1/3}\Phi'^2(y) + 2(\zeta' + \tau)\Phi(y)\Phi'(y)\Big], (\omega << \omega').$$

where $\overline{\kappa} = \kappa\upsilon\overline{\upsilon}$ and the argument of Airy function is determined at $\delta = 0$.

The integrands in (52) determine the differential cross section $d^2\sigma^{(\pm)}/d\upsilon d\tau$. The asymmetric dependence on the angular variable τ and the spin variables ζ and ζ' is due to P and C nonconservation in weak interactions and to the choice of kinematical conditions – an ultrarelativistic charged leptons and ultrarelativistic positron are emitted at small angles (not greater than ($\leq \gamma^{-1}$) with respect to the direction of the high-energy-(anti) neutrino momentum.

The range of the spectral variable υ is determined from the condition $\lambda\bar{\upsilon} - 1 \geq 0$, which yields

$$0 \leq \upsilon \leq \upsilon_1 = 1 - \frac{1}{\lambda} \tag{53}$$

In the absence of a field, the considered reaction has a threshold; that is, the kinematically allowed region is

$$\lambda > 1 \tag{54}$$

The external-field effect on the process allowed in the absence of a field as well as determined by the parameter

$$\eta = \frac{\kappa}{|\lambda - 1|}. \tag{55}$$

Let us consider the asymptotic behavior of the total cross section for the process. At $\kappa << 1$ and $\lambda > 1$, we obtain the asymptotic expressions

$$\sigma^{(+)}(\zeta', \zeta) = \frac{G_F^2}{4\pi} m_\mu^2 \left[F_0(\lambda) + \left(\frac{\kappa}{\lambda - 1} \right)^2 F_+(\lambda) \right], \quad \omega >> \omega',$$

$$\sigma^{(-)}(\zeta', \zeta) = \frac{G_F^2}{4\pi} m_\mu^2 \left[F_0(\lambda) - 4\frac{\kappa}{\lambda - 1}\left(1 - \frac{1}{\lambda}\right)^2 \zeta' + \left(\frac{\kappa}{\lambda - 1} \right)^2 F_-(\lambda) \right], \tag{56}$$
$$\omega << \omega',$$

where

$$F_0(\lambda) = \frac{1}{3}(2\lambda + 1)\left(1 - \frac{1}{\lambda}\right)^2,$$

$$F_+(\lambda) = -\frac{8}{3}\left(1 + \frac{3}{\lambda^2} - \frac{2}{\lambda^3}\right), \tag{57}$$

$$F_{-}(\lambda) = -\frac{8}{3}\left(1-\frac{2}{\lambda}\right)\left(1-\frac{1}{\lambda}\right)^2 .$$

The external-field effect on the process is described by the parameter $\frac{\kappa}{\lambda - 1}$. This effect is stronger for polarized particles than for unpolarized particles, being of the first and of the second order in κ, respectively. In a relatively weak field $(\kappa << 1)$ such that the relation $\kappa \geq \lambda - 1$ nevertheless holds, the cross section for the process differs markedly from the cross section for the free process, the latter being very small near the threshold. For $\omega >> \omega'$ ($\omega << \omega'$) we predominantly have the generation of charged leptons (positrons) whose spins are aligned with (opposite to) the direction of the magnetic field **H**- that is $\zeta = +1$ ($\zeta = -1$). This effect is similar to the Sokolov-Ternov effect, the radiative polarization of electrons in a magnetic field due to synchrotron radiation.

For $\lambda < 1$, the free process (at $\kappa = 0$) is forbidden. In a weak field $(\eta << 1)$, the cross section for the process is exponentially small, which is characteristic of all processes that have a threshold in the absence of a field. In this case

$$\sigma \sim \kappa \exp\left(-\sqrt{3}\,\delta \frac{1-\lambda}{\kappa}\right). \tag{58}$$

For $\eta >> 1$ and $\kappa >> 1$ (strong field), we obtain the strong field asymptotic expressions for the cross sections

$$\sigma^{(+)}(\zeta',\zeta) = \frac{G_F^2}{4\pi^3} m_\mu^2 c_2 (3\kappa)^{2/3}, \quad \omega >> \omega' \quad , \tag{59}$$

$$\sigma^{(-)}(\zeta',\zeta) = \frac{G_F^2}{4\pi^3} m_\mu^2 \left[c_2 (3\kappa)^{2/3} - c_1 (3\kappa)^{1/3} \zeta'\right], \quad \omega << \omega' ,$$

where

$$c_1 = \frac{2}{5}\Gamma^4\left(\frac{1}{3}\right), \quad c_2 = \frac{15}{14}\Gamma^4\left(\frac{2}{3}\right).$$

We see that, for $\omega >> \omega'$ ($\omega' >> \omega$) there is a predominant production of positrons (charged leptons) polarized in the direction parallel (antiparallel) to the external field **H**.

Assuming that relic (anti)neutrinos are massless, we estimate their energy (that is the temperature of relic radiation) at $\omega' \sim 2K \simeq 1.7\times10^{-4}\, eV$. Our results are valid in the region of cosmic-neutrino energies,

$$\omega \le \frac{m_\mu^2}{\omega'} \le 10^{20} \quad \text{эВ}. \tag{60}$$

For the field parameter, we have

$$\kappa < 10^{-7}\left(\frac{H}{1\text{Гс}}\right). \tag{61}$$

The dipole fields of neutron stars are $H \le 10^{13}$. For this fields $\kappa \le 10^{6}$ and the cross section for the considered process in a magnetic field is much larger than the cross section for the free process owing to the factor

$$F(\kappa) = (3\kappa)^{2/3} \le 10^{4}. \tag{62}$$

Thus, the discussed process can be a source of high-energy charged leptons; in the vicinity of strongly magnetized stars, their spectral distributions and total fluxes can differ considerably from the corresponding values in the regions where the field can be disregarded. Neutron stars may modify sizably the energy spectra of cosmic rays owing to the large-scale pulsar-wind effect.

4. Determination of the Weinberg angle in a magnetic field

It is well known that the relation between the weak coupling constant *g* and the electric charge *e* is established with the Weinberg angle θ_w. Generally, the value

$$\sin^2\theta_w = \frac{e^2}{g^2} \tag{63}$$

is not constant. It is a function of the momentum transfer squared. It is known that $\sin^2\theta_w$ can be determined by measuring the ratio of the cross sections of the reactions

$$\nu_l + e^- \to \nu_l + e^-, \tag{64}$$

$$\bar{\nu}_l + e^- \to \bar{\nu}_l + e^- \tag{65}$$

where $\nu_l = \nu_\mu, \nu_\tau$ and $\bar{\nu}_l = \bar{\nu}_\mu, \bar{\nu}_\tau$.

In this section we present the results of our calculations on $\sin^2\theta_w$ in an external magnetic field in the framework of the Weinberg-Salam model.

In the framework of the Weinberg-Salam model the cross sections of the considered processes in a constant external magnetic field are

$$\sigma_\pm = \frac{G_F^2 m_e^2}{\pi^{2/3}} \int_0^\infty \frac{u\,du}{(1+u)^4} \left\{ \left[\frac{\kappa}{u} f_\pm - 2 g_L g_R (1+u) \right] \Phi_1(t) - 2\left(\frac{\chi}{u}\right)^{2/3} f_\pm \Phi'(t) \right\} \left[1 + \kappa \left(\frac{m_e}{m_z}\right)^2 \frac{u}{1+u} \right]^{-2} \tag{66}$$

where σ_+ and $f_+ = g_L^2(1+u)^2 + g_R^2$ are for the first reaction, σ_- and $f_- = g_R^2(1+u)^2 + g_L^2$ are for the second reaction, $g_L = -\frac{1}{2} + \sin^2\theta_w$, $g_R = \sin^2\theta_w$. Here the functions

$$\Phi'(t) = \frac{d\Phi(t)}{dt} \tag{67}$$

and

$$\Phi_1(t) = \int_t^\infty \Phi(\rho)\,d\rho \tag{68}$$

are determined with the Airy function

$$\Phi(t) = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{\infty} du \exp\left[i(ty + \frac{1}{3} y^3) \right]. \tag{69}$$

All these functions depend on a new variable

$$t = \left(\frac{u}{\chi}\right)^{\frac{2}{3}} \left(1 - \frac{\kappa}{u}\right). \tag{70}$$

To determine $\sin^2\theta_w$ in an external magnetic field we find the ratio $R = \sigma_+/\sigma_-$:

$$R = \frac{A_0 \sin^4\theta_W - (A_0 - B_0)\sin^2\theta_W + H_0 + \frac{2}{s^3}\eta^2\left[D_0 \sin^4\theta_W - (D_0 - E_0)\sin^2\theta_W + I_0\right]}{A_0 \sin^4\theta_W - B_0 \sin^2\theta_W + C_0 + \frac{2}{s^3}\eta^2 (D_0 \sin^4\theta_W - E_0 \sin^2\theta_W + G_0)} \qquad (71)$$

where

$$A_0 = a + b - c = 4s^2 - 2s + 1\,, \qquad (72)$$

$$B_0 = b - \frac{1}{2}c = s^2 - \frac{1}{2}s + 1\,, \qquad (73)$$

$$C_0 = \frac{1}{4}b = \frac{1}{4}(s^2 + s + 1)\,, \qquad (74)$$

$$D_0 = d + f - h = -2(s^4 - 6s^2 + 8s - 5)\,, \qquad (75)$$

$$E_0 = f - \frac{1}{2}h = -s^3 + 3s^2 - 10s + 10\,, \qquad (76)$$

$$G_0 = \frac{1}{4}f = \frac{1}{4}(-3s^2 - 4s + 10)\,, \qquad (77)$$

$$H_0 = \frac{1}{4}a = \frac{3}{4}s^2\,, \qquad (78)$$

$$I_0 = \frac{1}{4}d = \frac{1}{4}s^2(-2s^2 + 2s + 3) \qquad (79)$$

and

$$a = 3s^2\,, \qquad (80)$$

$$b = s^2 + s + 1\,, \qquad (81)$$

$$c = 3s\,, \qquad (82)$$

$$d = s^2(-2s^2 + 2s + 3)\,, \qquad (83)$$

$$f = -3s^2 - 4s + 10\,, \qquad (84)$$

$$h = 2s(s^2 - 6s + 6)\,. \qquad (85)$$

In the expressions (71-85) $s = \kappa + 1 = (k+p)^2 / m_e^2$ is the normalized Mandelstam variable.

Solving the corresponding equation derived from the expression (71) we find the general formula for $\sin^2\theta_w$ in a weak magnetic field

$$\sin^2\theta_w = \frac{1}{2(R-1)\left(1 + 2\frac{\eta^2}{s^3}\frac{D_0}{A_0}\right)}\left\{(R+1)\frac{B_0}{A_0} - 1 + 2\frac{\eta^2}{s^3}\left[(R+1)\frac{E_0}{A_0} - \frac{D_0}{A_0}\right] \mp\right.$$

$$\mp\left\{\left[(R+1)\frac{B_0}{A_0}-1\right]^2+4(R-1)\left(\frac{H_0}{A_0}-R\frac{C_0}{A_0}\right)+4\frac{\eta^2}{s^3}\times\right.$$

$$\left.\times\left[\left[(R+1)\frac{B_0}{A_0}-1\right]\left[(R+1)\frac{E_0}{A_0}-\frac{D_0}{A_0}\right]+2(R-1)\left[\frac{I_0}{A_0}-R\frac{G_0}{A_0}+\frac{D_0}{A_0}\left(\frac{H_0}{A_0}-R\frac{C_0}{A_0}\right)\right]\right]\right\}^{\frac{1}{2}}. \quad (86)$$

If we take into account $s=\kappa+1$ and the definition of the parameter

$$\eta=\frac{\chi}{\kappa}, \quad (87)$$

we can see that $\sin^2\theta_w$ depends on the kinematic parameter

$$\kappa=\frac{2(kp)}{m_e^2}=\frac{2k_0\varepsilon}{m_e^2} \quad (88)$$

and the dynamic parameter

$$\chi=\frac{e}{m_e^3}\left[-(F_{\alpha\beta}p^\beta)^2\right]^{\frac{1}{2}}=\frac{\varepsilon}{m_e}\frac{H}{H_0}, \quad (89)$$

where ε is the energy of the initial electron and m_e is the electron mass, $F_{\alpha\beta}$ is the tensor of the constant external electromagnetic field, p is the 4-momentum of the initial electron, k is 4-momentum of the initial neutrino (antineutrino), k_0 is the energy of the initial neutrino (antineutrino), H is the strength of an external magnetic field, H_0 is the Schwinger field strength, e is the elementary charge.

. The dependence of $\sin^2\theta_w$ on the dynamic parameter χ, *i.e.* on the strength of the external magnetic field rouses interest.

When $H=0$, the parameter $\eta=0$ and we derive the formula for $\sin^2\theta_w$ for the
free case

$$\sin^2\theta_w=\frac{(R+1)\frac{B_0}{A_0}-1\mp\sqrt{\left[(R+1)\frac{B_0}{A_0}-1\right]^2+4(R-1)\left(\frac{H_0}{A_0}-R\frac{C_0}{A_0}\right)}}{2(R-1)}. \quad (90)$$

And now let us evaluate the contribution of an external magnetic field to $\sin^2\theta_w$. From the expression (86) we see that the contribution $\Delta\sin^2\theta_w$ to $\sin^2\theta_w$ is determined by the terms proportional to η^2. To evaluate $\Delta\sin^2\theta_w$ let us look at one of these terms:

$$\Delta\sin^2\theta_w \sim \left|\frac{1}{(R-1)}\frac{\eta^2}{s^3}\frac{D_0}{A_0}\right| = \frac{1}{2(R-1)}\frac{\eta^2}{s}\frac{1-\frac{6}{s^2}+\frac{8}{s^3}+\frac{5}{s^4}}{1-\frac{1}{2s}+\frac{1}{4s^2}}. \qquad (91)$$

If we set $H = 10^{-2} H_0$, $\varepsilon = 0.1 GeV$, $R \to 3$ (low-energy region) and suppose that the neutrino (antineutrino) mass is zero or very small (tens or hundreds eV) and it has very low energy (for example, $k_0 = 10 KeV$), we have $s = 8.689$ and $\Delta\sin^2\theta_w \approx 1.83\times10^{-3}$. In this case the evaluations give $\approx 1\%$ for $\Delta\sin^2\theta_w / \sin^2\theta_w$.

The results obtained in this work are right for ν_μ, $\bar{\nu}_\mu$, ν_τ and $\bar{\nu}_\tau$.

Acknowledgements

We are very greatful to Franco Bradamante, Anna Martin and the organizing committee of SPIN 2004 Symposium for their invitation and warm hospitality. We also would like to thank Vladimir Zhukovskii, Anatolii Borisov for many useful discussions. We also appreciate the valuable conversations with many participants of the symposium very much. This work is supported in part by Tempus Individual Mobility Grant.