Calculations of single-inclusive cross sections and spin asymmetries in pp scattering

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Outline:

Talk will focus on general status & progress of calculations, rather than on specific predictions for spin asymmetries etc.

- Introduction
- Next-to-leading order
- Resummation
- Power corrections
- Conclusions

I. Introduction

High- p_T single-inclusive reactions :

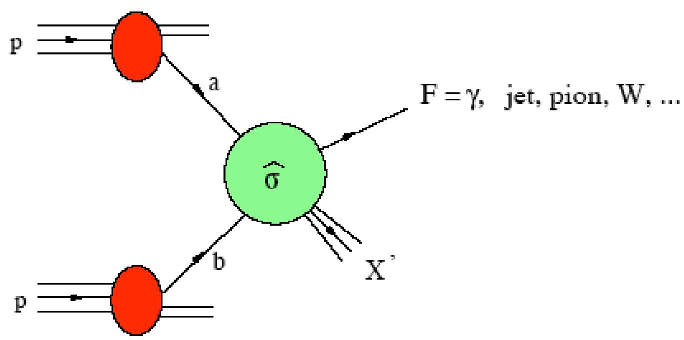
$$\mathrm{pp} \to \pi\, \mathrm{X}\,, \quad \mathrm{pp} \to \mathrm{jet}\, \mathrm{X}\,, \quad \mathrm{pp} \to \gamma\, \mathrm{X}\,, \quad \ldots$$

- **the "next-simplest" observables in pp scattering, after Drell-Yan**
- **short-distance interaction → perturbation theory**
- **important tests of QCD hard scattering, need for higher orders, resummations, power corrections**
- **important insights into proton structure, and into formation of hadronic final states**

One prime example: $\vec{\mathrm{p}}\vec{\mathrm{p}}$ at RHIC, to measure $\Delta \mathrm{g}$

Hard scattering in hadron collisions

$$p_T^3 \frac{d\sigma}{dp_T d\eta} = \left| \vphantom{\frac{a}{b}} \right.$$



$$\left. \vphantom{\frac{a}{b}} \right|^2 + \mathcal{O}\left(\frac{\lambda}{p_T}\right)^n$$

$$p_T^3 \frac{d\sigma^{pp\to \mathrm{F}X}}{dp_T d\eta} = \sum_{abc} \int dx_a\, dx_b\, f_a(x_a,\mu)\, f_b(x_b,\mu) \times p_T^3 \frac{d\hat{\sigma}^{ab\to \mathrm{F}X'}}{dp_T d\eta}(x_a P_a, x_b P_b, P^{\mathrm{F}}, \mu) \;+\; \text{P.C.}$$

$\hat{\sigma}^{(0)} + \alpha_s \hat{\sigma}^{(1)} + \ldots$ perturb.

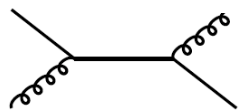
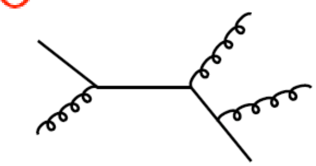
(for pions, additional fragmentation functions)

Ingredients for theoretical framework:

- **parton distributions / fragmentation functions**

- **higher order QCD corrections**

$$\hat{\sigma} = \underbrace{\hat{\sigma}^0}_{\text{LO}} + \underbrace{\alpha_s \hat{\sigma}^1}_{\text{NLO}} + \ldots$$

  * **may be particularly sizable in hadronic collisions**
  * **reduction in scale dependence**
  * **NLO = state of the art**
  * **in some cases, all-order resummations of dominant terms**

- **idea about power corrections, e.g., if p_T not so high**

II. NLO - successes and failures

NLO calculations for single-inclusive processes :

$\mathrm{pp} \rightarrow \gamma \mathrm{X}$

Aurenche et al. ; Baer, Ohnemus, Owens ;
Contogouris et al. ; Gordon, WV ;
Mukherjee, Stratmann, WV

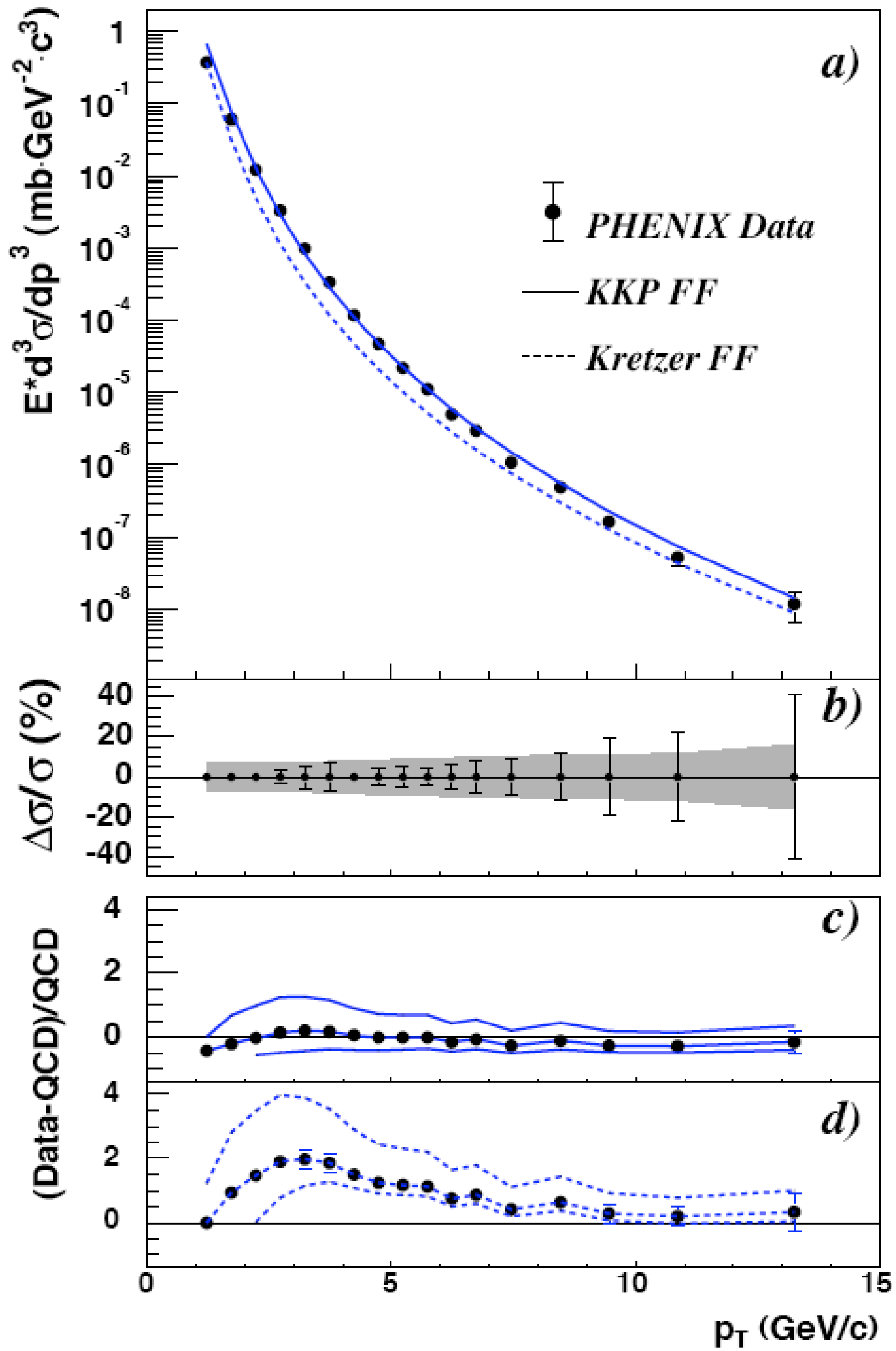
$\mathrm{pp} \rightarrow \mathrm{jet}\ \mathrm{X}$

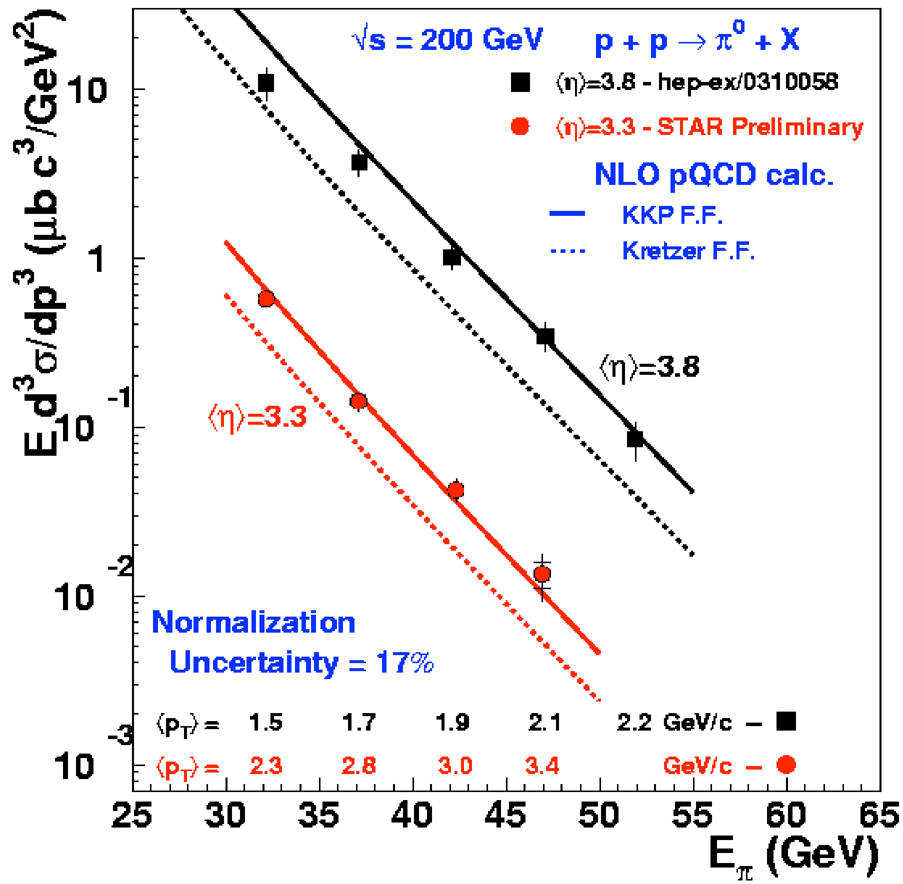
Ellis, Kunszt, Soper; Furman ; Guillet ; …
De Florian, Frixione, Signer, WV ;
Jäger. Stratmann, WV

$\mathrm{pp} \rightarrow \mathrm{h}\ \mathrm{X}$

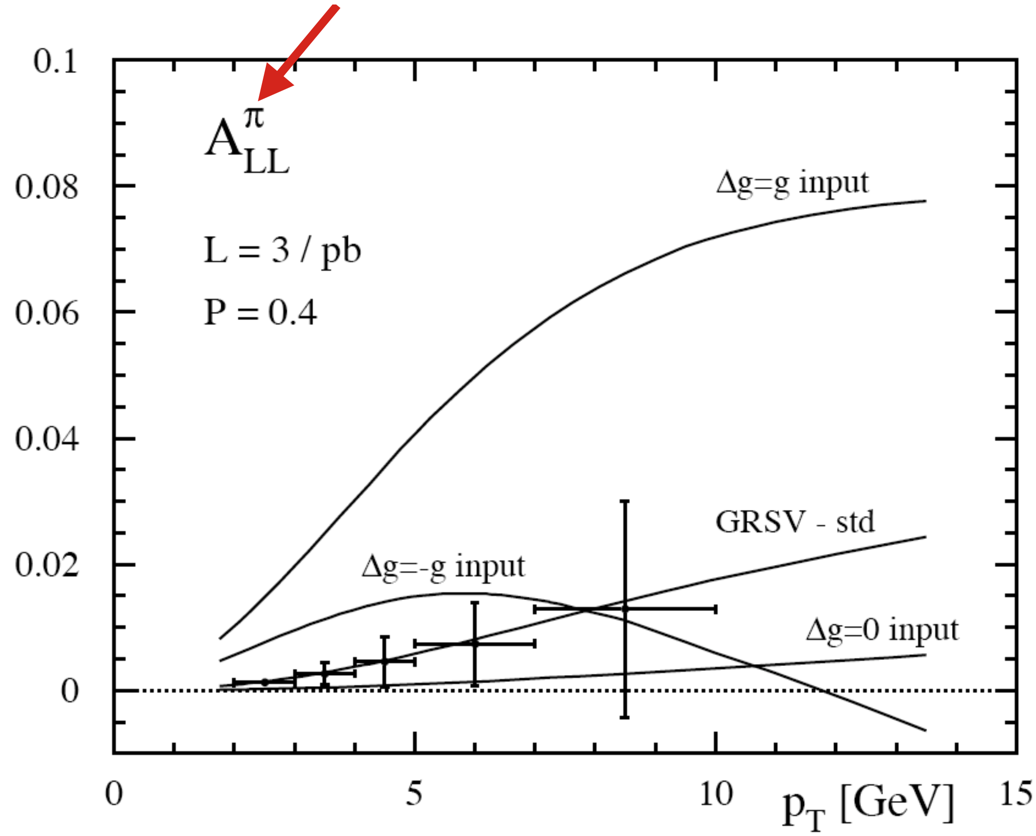
Aversa et al.; de Florian;
Jäger, Schäfer, Stratmann, WV

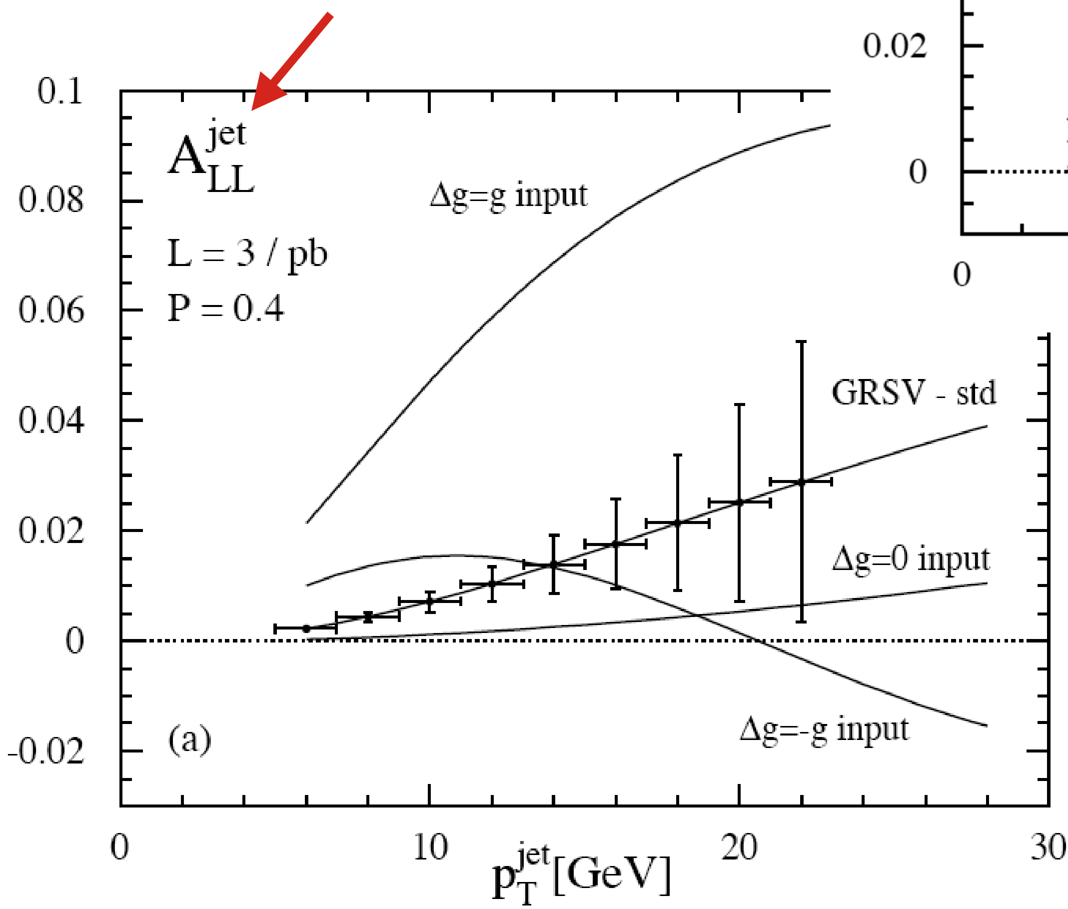
$pp \rightarrow \pi^0 X$ at RHIC



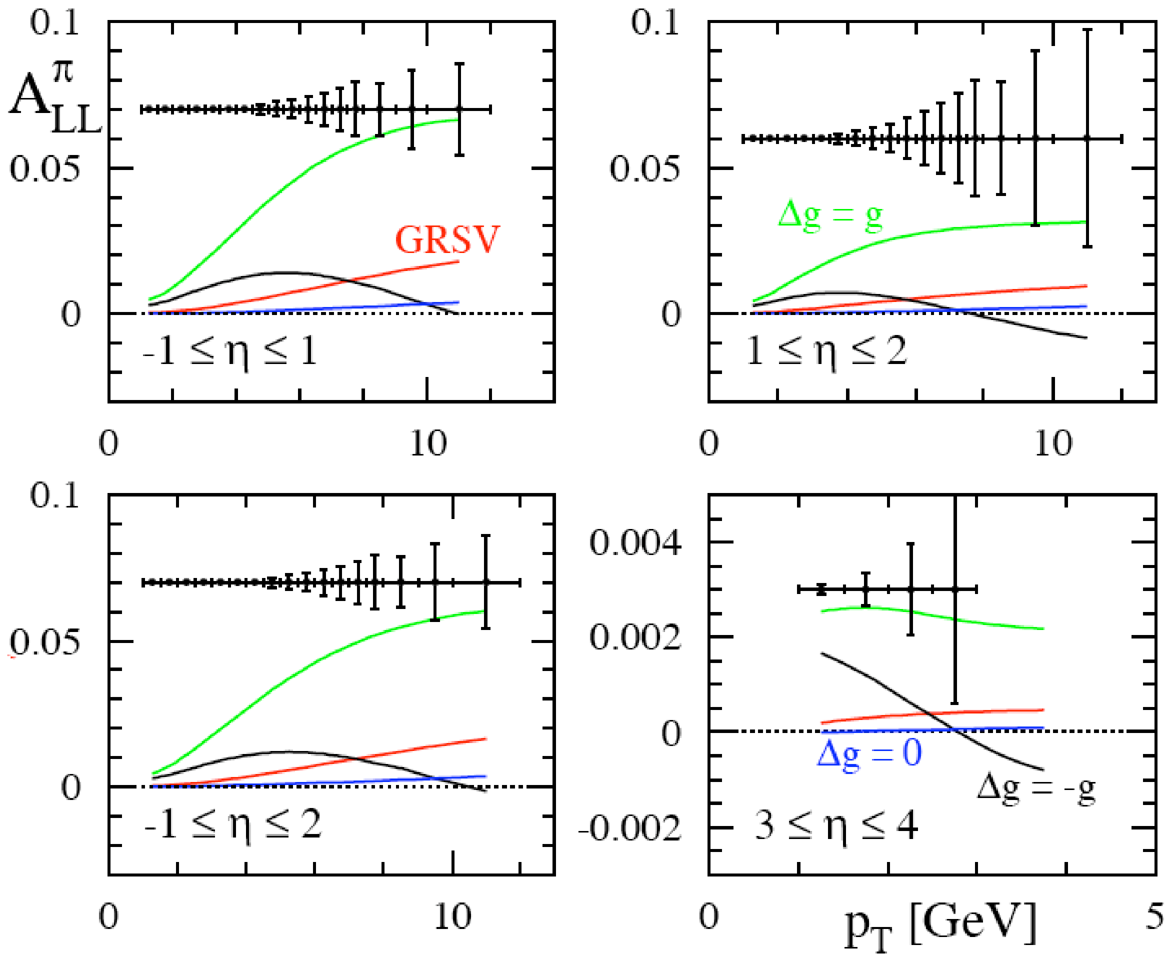


Spin asymmetries at RHIC





(Stratmann, Jäger, WV)

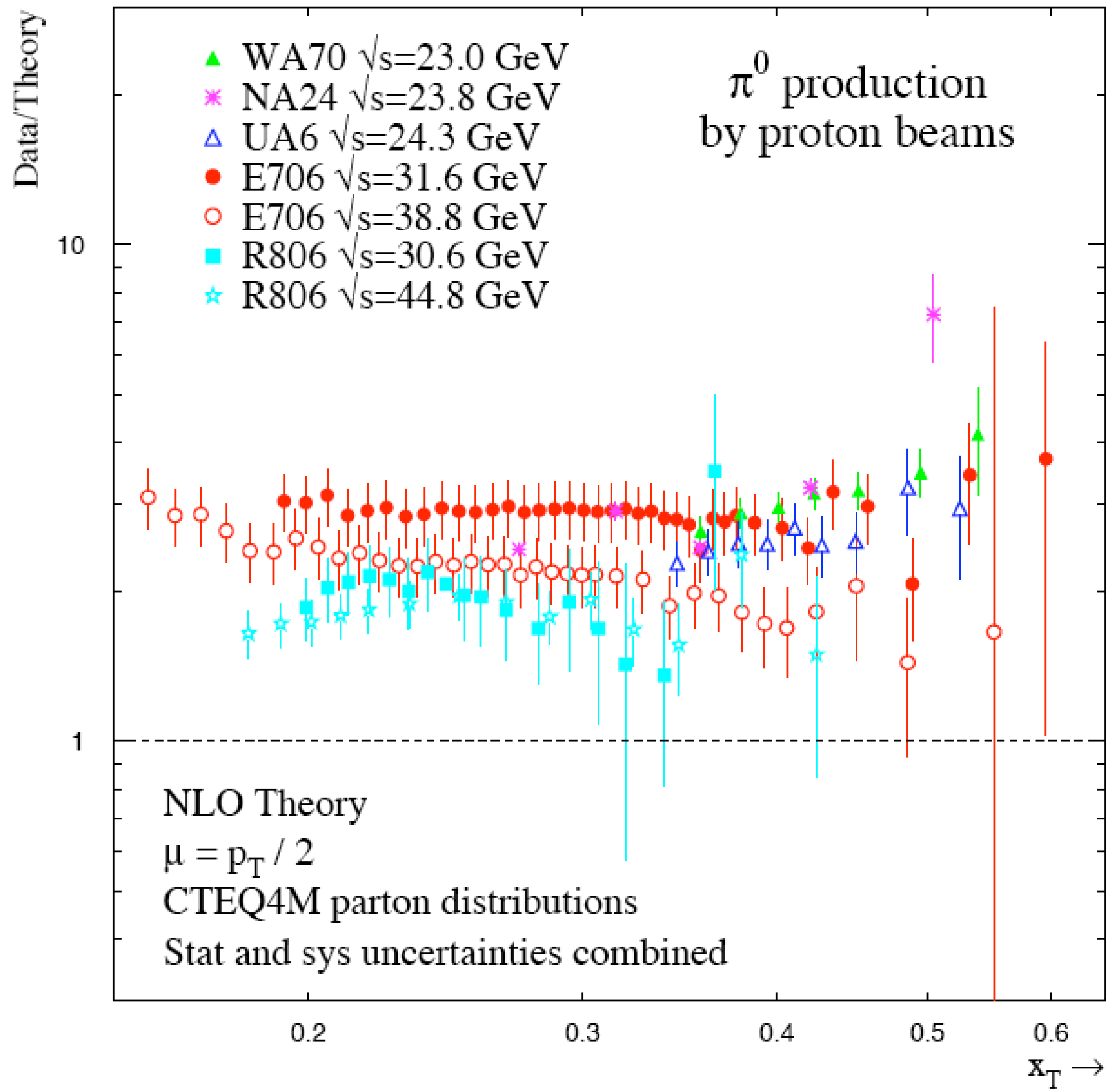


(Stratmann, Jäger, WV)

(L = 7/pb, P = 0.4)

$pp \to \pi^0 X$

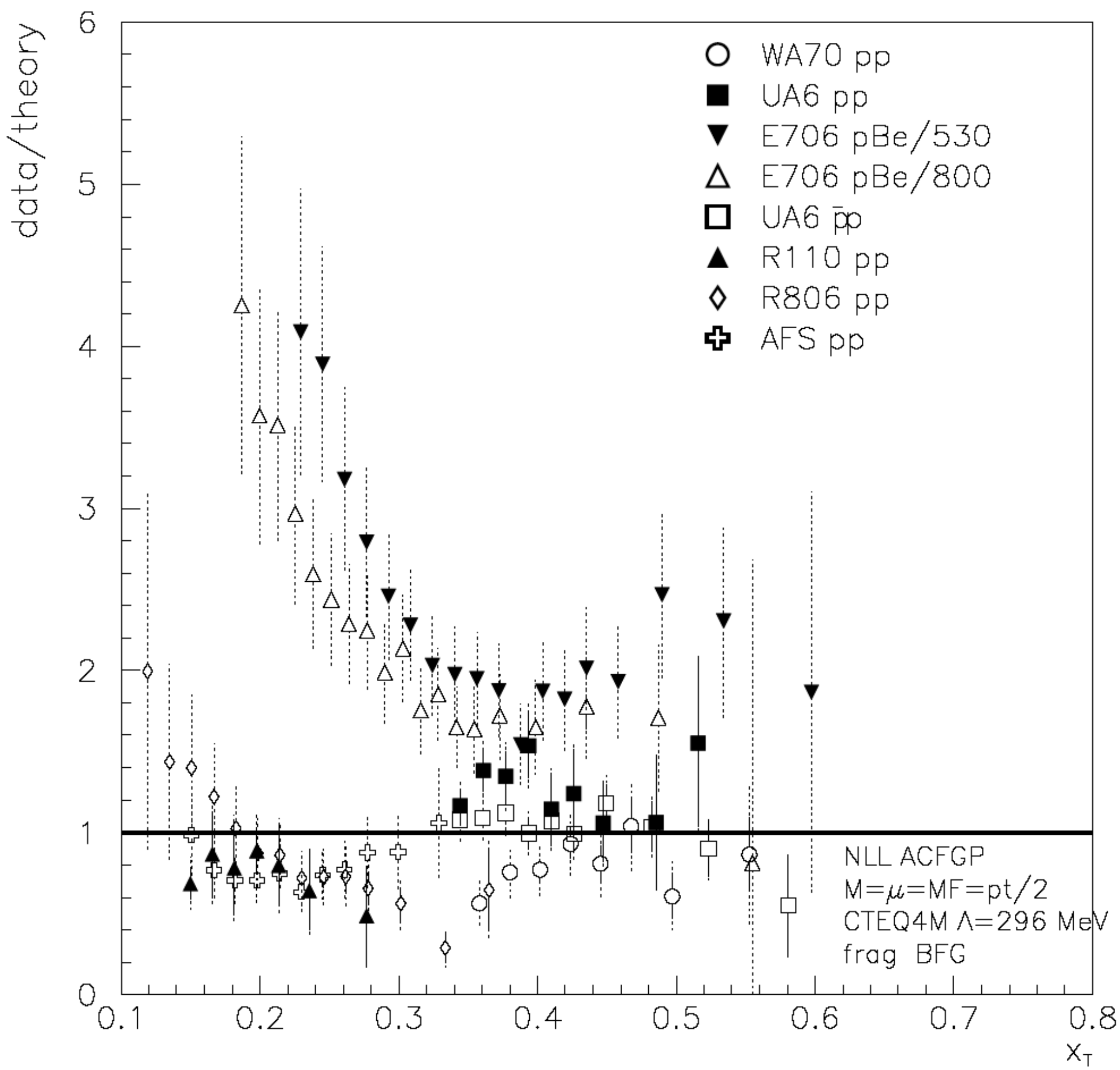
at lower energies



Apanasevich et al.
(see also Aurenche et al.; Bourrely, Soffer)

$pp \to \gamma X$ at lower energies

Aurenche et al.



→ **Why problems in fixed-target regime ?**

Why "near perfect" at colliders ?

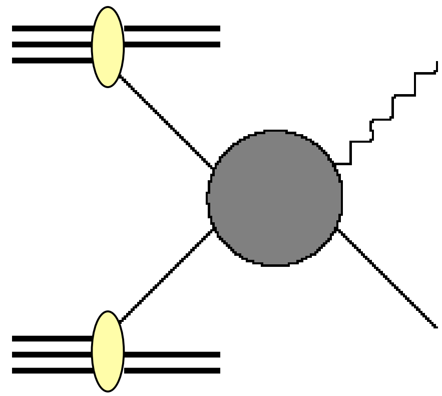
→ • **higher-order corrections beyond NLO ?**

• **power corrections ?**
(e.g., "intrinsic" k_T as a model. Actually tries to mimic higher orders and power corrections. There is no unique way of implementing it. See Murgia's talk)

III. Resummation

Threshold resummation

Example: prompt photons $\mathbf{pp \to \gamma X}$



$$\hat{\mathbf{x}}_{\mathbf{T}} = \frac{\mathbf{2p_T}}{\sqrt{\hat{\mathbf{s}}}} \to \mathbf{1}$$

$$p_T^3 \frac{d\hat{\sigma}_{ab}}{dp_T} = p_T^3 \frac{d\hat{\sigma}_{ab}^{\mathrm{Born}}}{dp_T} \Big[1 + \underbrace{\mathcal{A}_1 \alpha_s \ln^2\left(1-\hat{x}_T^2\right) + \mathcal{B}_1 \alpha_s \ln\left(1-\hat{x}_T^2\right)}_{\text{NLO}}$$

$$+ \ldots + \mathcal{A}_k \alpha_s^k \ln^{2k}\left(1-\hat{x}_T^2\right) + \ldots \Big] + \ldots$$

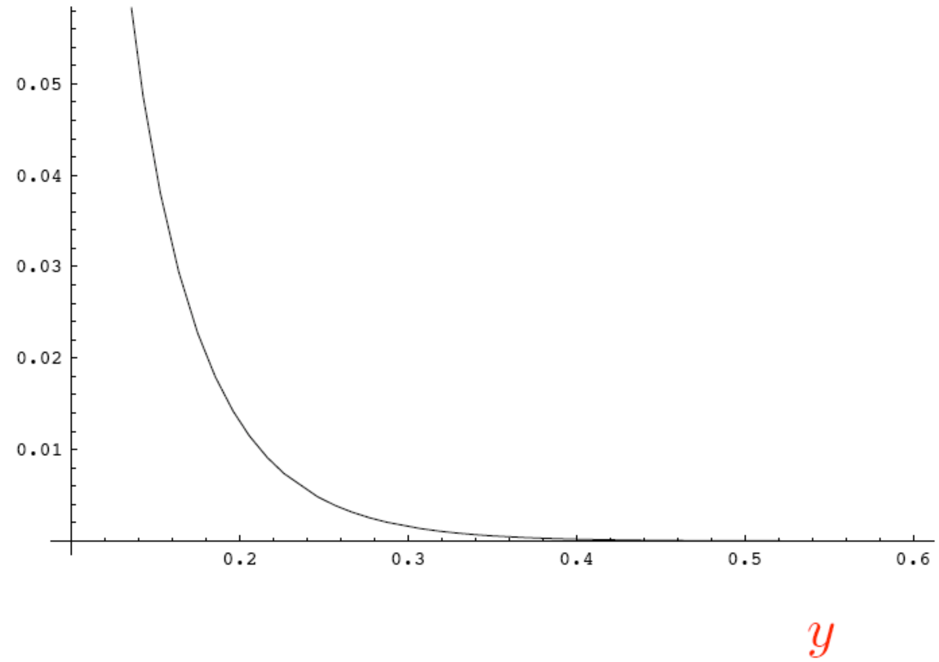
"threshold" logarithms, associated with soft/coll. gluons

Are these terms relevant ?

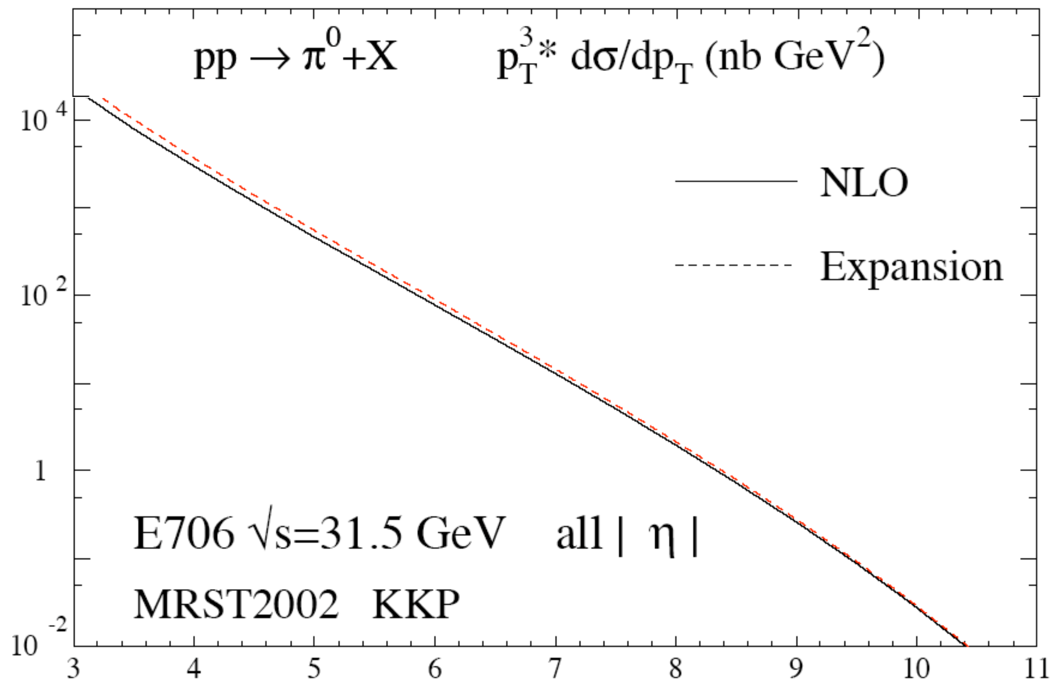
$$\frac{d\sigma}{dp_\perp} = \sum_{a,b} \int_{x_T^2}^{1} \frac{dy}{y} \, \mathcal{L}_{ab}(y) \, \frac{d\hat{\sigma}_{ab}\left(\frac{x_T^2}{y}\right)}{dp_\perp}$$

↑

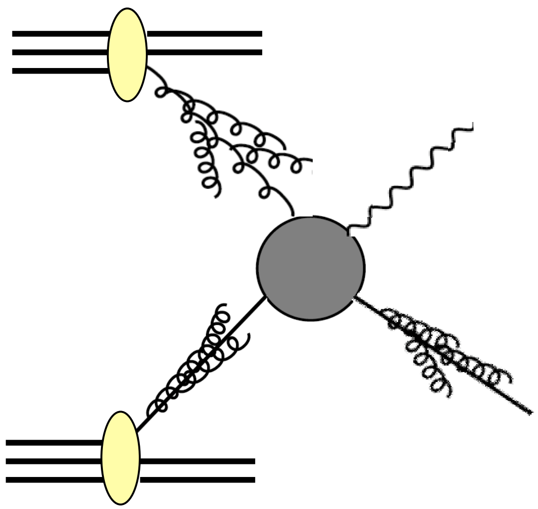
"parton luminosity" $(f_a \otimes f_b)(y)$



How dominant are they ?



Large terms may be resummed to all orders in α_s



Sterman; Catani, Trentadue;

Laenen, Oderda, Sterman;

Catani, Mangano, Nason, Oleari, WV;

Sterman, WV; Kidonakis, Owens

- exponentiation occurs when Mellin moments in $\hat{x}_T^2$ are taken

$$\alpha_s^k \, \ln^{2k}(1-\hat{x}_T^2) \;\rightarrow\; \alpha_s^k \, \ln^{2k}(N)$$

Leading logarithms $\alpha_s^k \ln^{2k}(N)$:

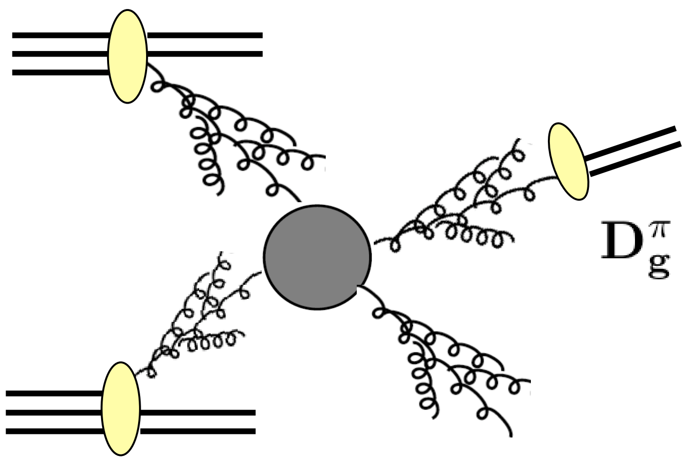
$$q\bar{q} \longrightarrow \gamma g \qquad \exp\left[\left(C_F + C_F - \frac{1}{2} C_A \right) \frac{\alpha_s}{\pi} \ln^2(N) \right]$$

$$qg \longrightarrow \gamma q \qquad \exp\left[\left(C_F + C_A - \frac{1}{2} C_F \right) \frac{\alpha_s}{\pi} \ln^2(N) \right]$$

- **these exponents are positive → enhancement**

(NLL far more complicated, known:
Laenen, Oderda, Sterman; Catani. Mangano, Nason)

Same for inclusive hadrons $pp \to \pi^0 X$



$gg \to gg$ $$\exp\left[\left(C_A + C_A + C_A - \frac{1}{2}C_A\right)\frac{\alpha_s}{\pi}\ \ln^2(N)\right]$$

- expect much larger enhancement !
- important techniques for NLL color exchange : Sen; Kidonakis, Sterman; Bonciani et al.
- also, use NLO calculation by Jäger, Schäfer, Stratmann, WV

- **What is the general structure ?** $\mathbf{L} \equiv \ln(\mathbf{N})$

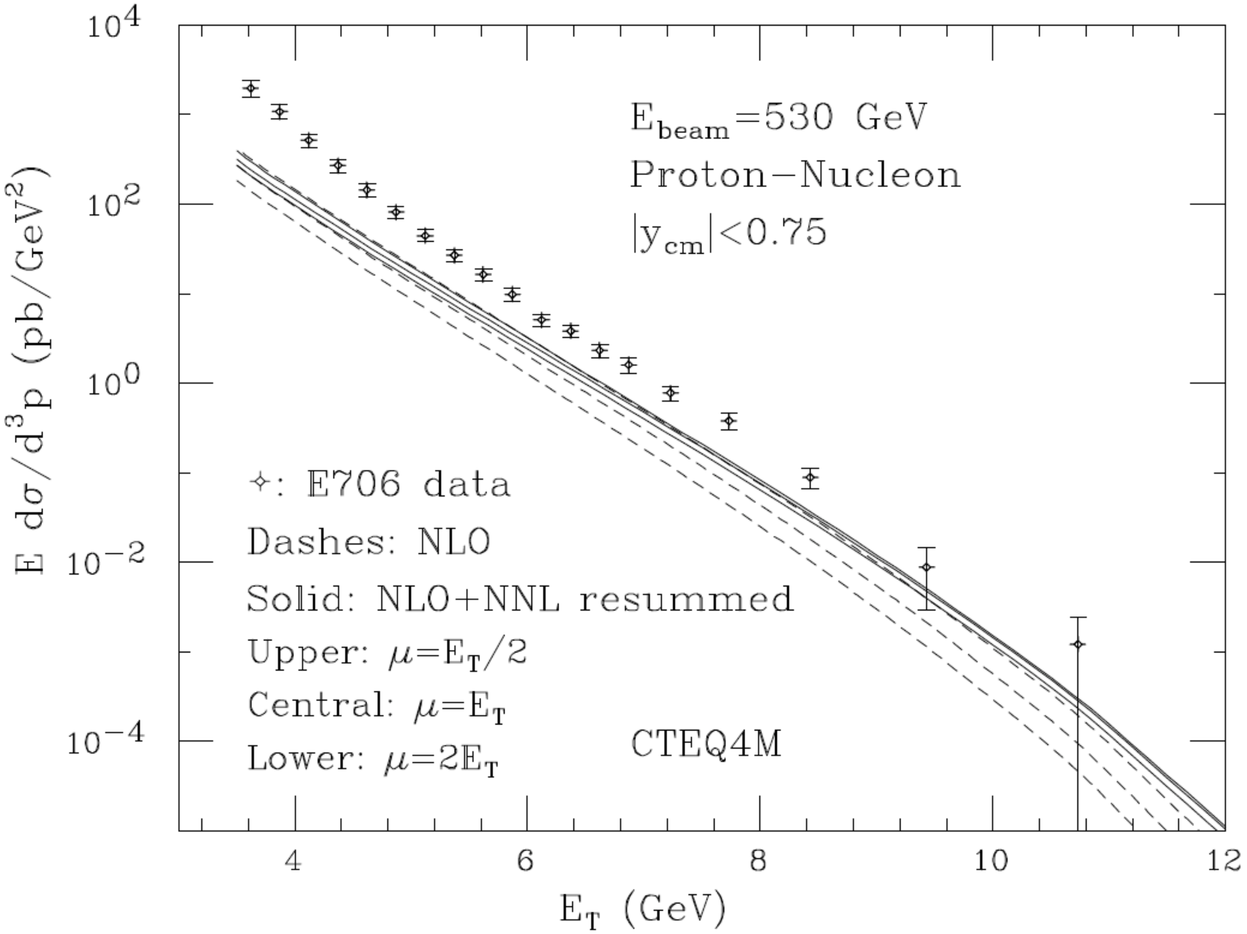
Fixed order →

Resummation ↓

LO	1				
NLO	$\alpha_s\, L^2$	$\alpha_s\, L$	α_s		$+ \ldots$
NNLO	$\alpha_s^2\, L^4$	$\alpha_s^2\, L^3$	$\alpha_s^2\, L^2$	$\alpha_s^2\, L$	$+ \ldots$
	$\alpha_s^3\, L^6$	$\alpha_s^3\, L^5$	$\alpha_s^3\, L^4$	$\alpha_s^3\, L^3$	$+ \ldots$
	$\alpha_s^4\, L^8$	$\alpha_s^4\, L^7$	$\alpha_s^4\, L^6$	$\alpha_s^4\, L^5$	$+ \ldots$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
N^kLO	$\alpha_s^k\, L^{2k}$	$\alpha_s^k\, L^{2k-1}$	$\alpha_s^k\, L^{2k-2}$	$\alpha_s^k\, L^{2k-3}$	$+ \ldots$
	LL	**NLL**	**NNLL**		

$pp \to \gamma X$

E706



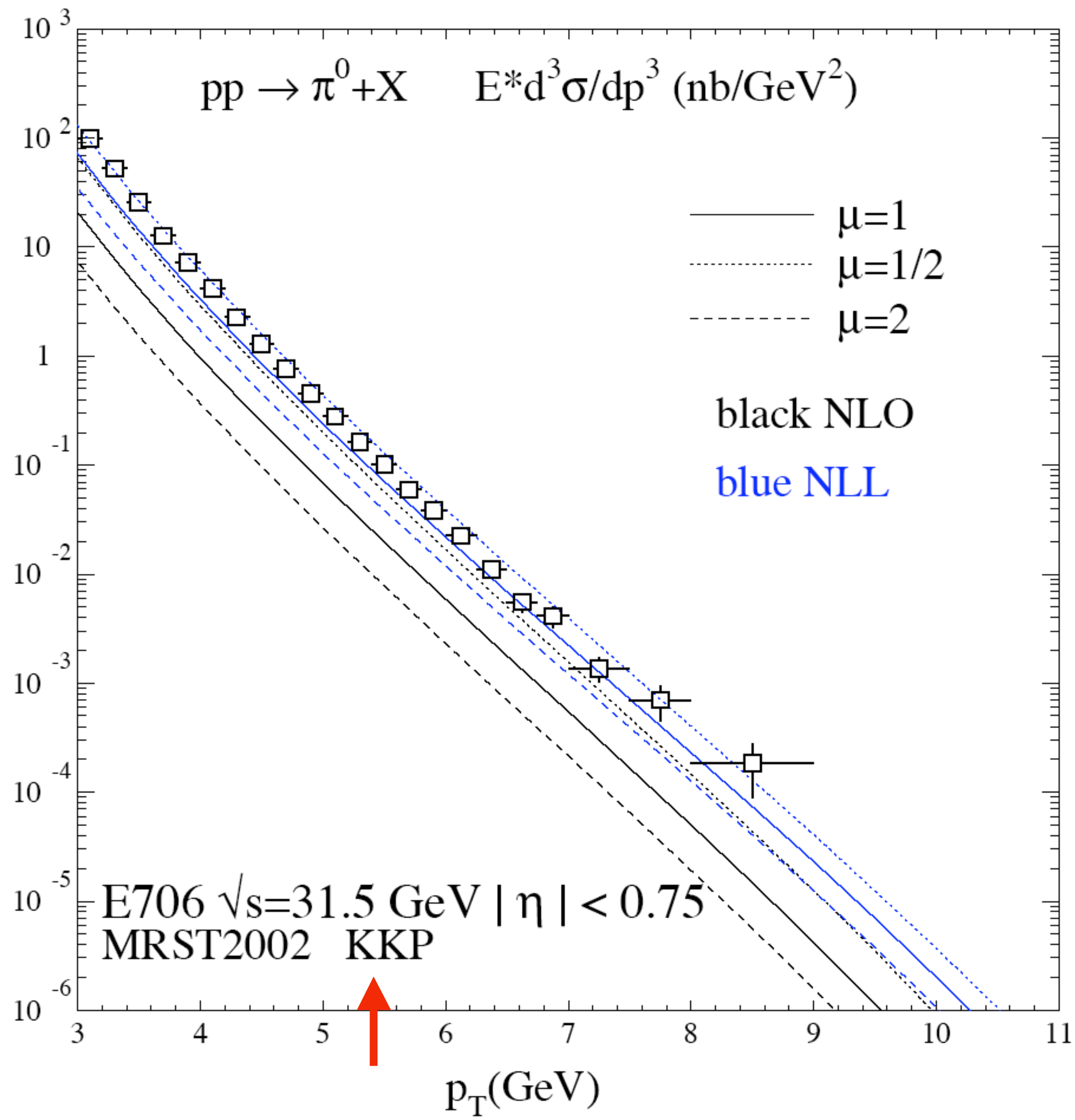
Catani,Mangano,Nason,Oleari,WV;

Sterman, WV

- ∗ **Significant reduction in scale dependence !**
  - **resummation compensates soft part of DGLAP evolution**
- ∗ **Effects negligible at collider energies**

E706

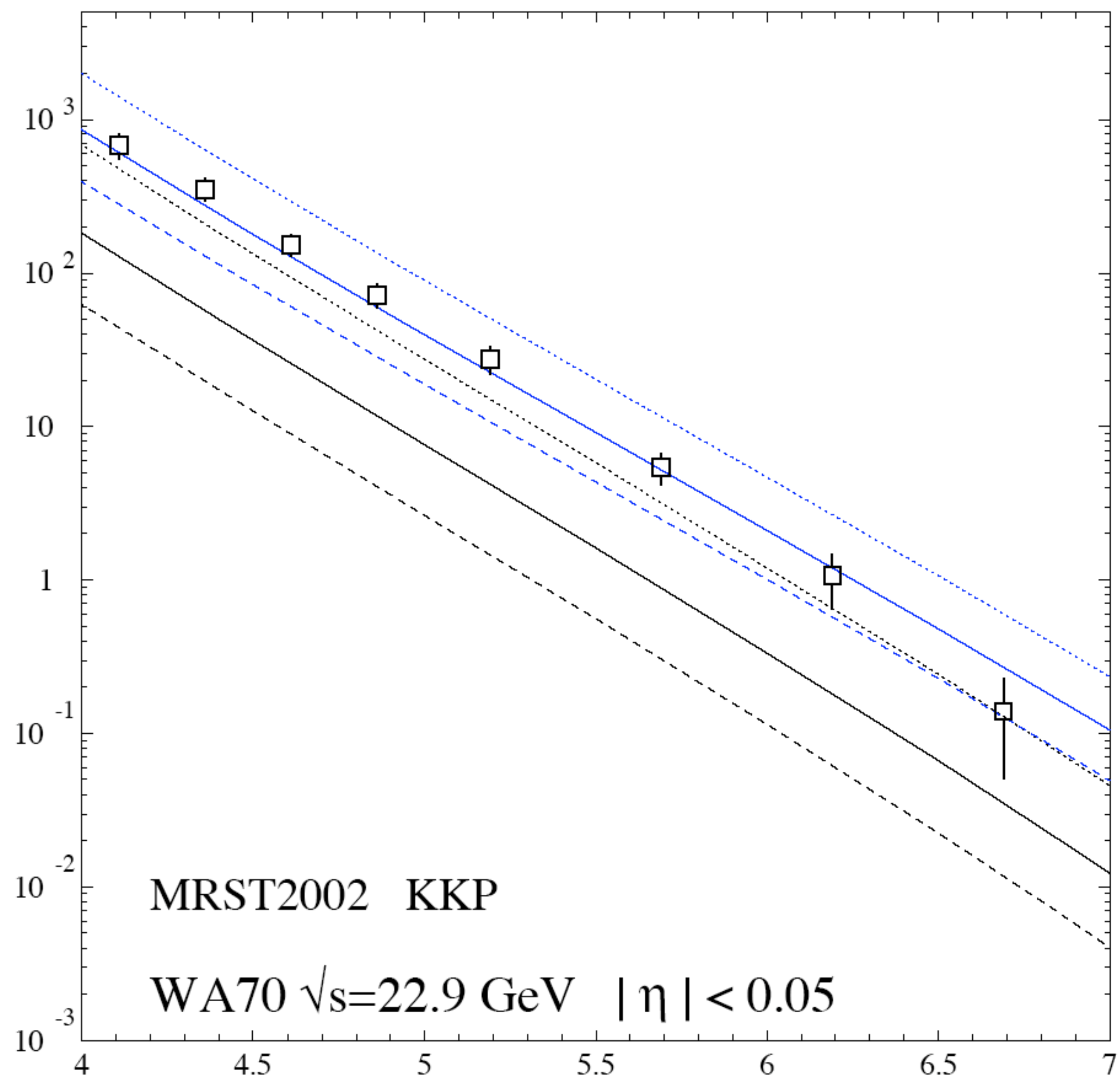
$pp \to \pi^0 X$



de Florian, WV

(note, the resummation is for rapidity-integrated cross section)

WA70



- **discrepancies for pion production are strongly reduced**
- **colliders: much smaller effects (but watch out for subleading terms)**
- **curious situation:**
  bigger problems between data and NLO for prompt γ
  however, resummation effects larger for pions !
- **experimental issues ?**
- **how about power corrections ?**

IV. Power Corrections

DY: $$\exp\left[\frac{2C_F}{\pi}\int_0^1 dy\,\frac{y^N-1}{1-y}\int_{Q^2}^{Q^2(1-y)^2}\frac{dk_\perp^2}{k_\perp^2}\,\alpha_s(k_\perp^2)\,+\,\ldots\right]$$

- ill-defined because of strong-coupling regime
- singularity in α_s reflected in factorial growth of PT series
- $\rightarrow$ structure of power corrections from resummation ?

$$\exp\left[\frac{2C_F}{\pi}\int_0^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\,\alpha_s(k_\perp^2)\,\left\{K_0\left(\frac{2Nk_\perp}{Q}\right)+\ln\left(\frac{Nk_\perp}{Q}\right)\right\}+\,\ldots\right]$$

- regime of very low $k_\perp$:

$$\exp\left[\frac{2C_F}{\pi}\frac{N^2}{Q^2}\int_0^{\lambda^2}dk_\perp^2\,\alpha_s(k_\perp^2)\,\ln\left(\frac{Q}{Nk_\perp}\right)\right]\sim\exp\left[\frac{2C_F}{\pi}\frac{N^2}{Q^2}\left\{g_1+g_2\ln\left(\frac{Q}{NQ_0}\right)\right\}\right]$$

- overall powers all even, exponentiating !

Sterman, WV

- **closer analysis shows that there are also effects from transverse-momentum resummation**
  connects g_1 and g_2 to values from k_T analysis ?

- **for single-inclusive cross sections** $pp \to \gamma X$

$$Q \leftrightarrow 2p_T$$

$$N \leftrightarrow \frac{1}{\ln x_T^2} \qquad x_T \equiv \frac{2p_T}{\sqrt{s}}$$

- **then,**

$$\frac{N^2}{Q^2} \sim \frac{1}{p_T^2 \ln^2\left(\frac{4p_T^2}{s}\right)}$$

- **qualitatively, for inclusive hadrons, replace** $p_T \to p_T/z$

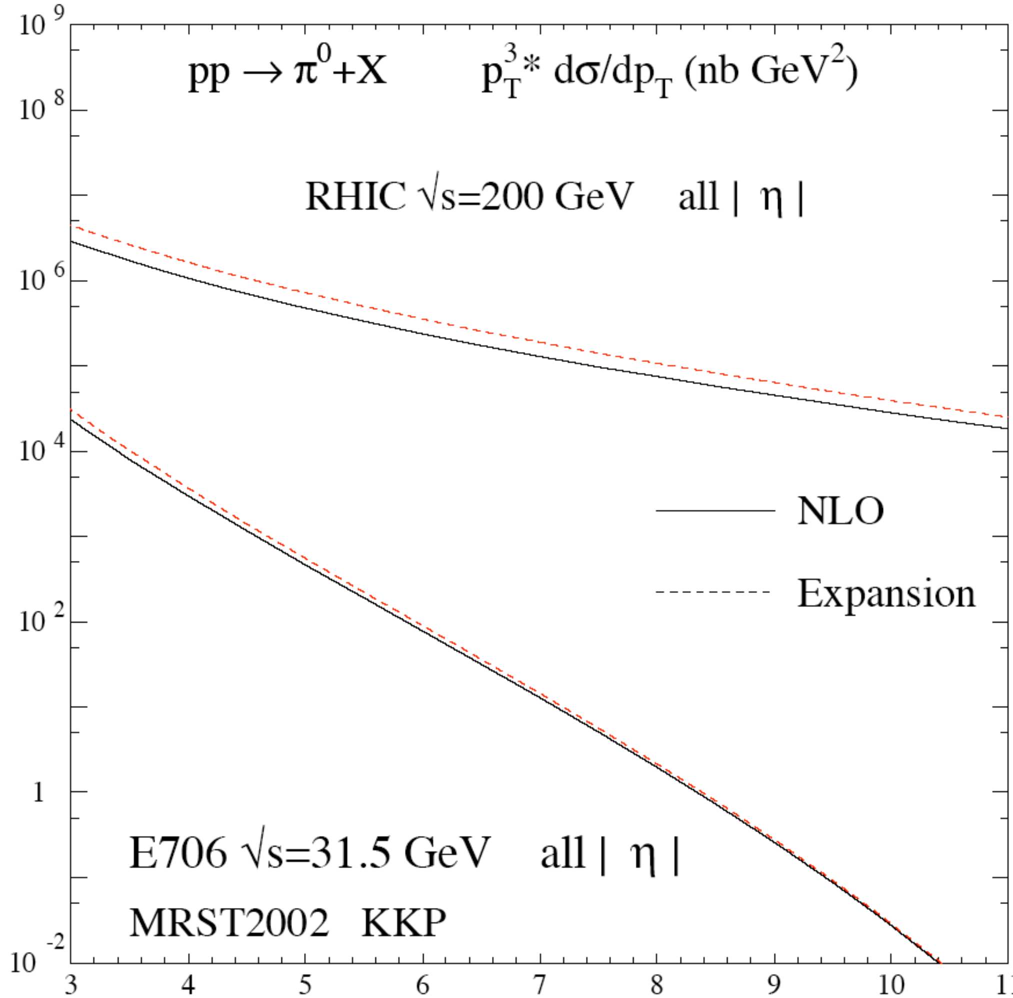
V. Conclusions

- **tremendous success of NLO at collider energies**
- **at lower energies: resummation appears crucial**
  large effects for inclusive hadrons
  reduction of scale dependence
  will be relevant in many situations !

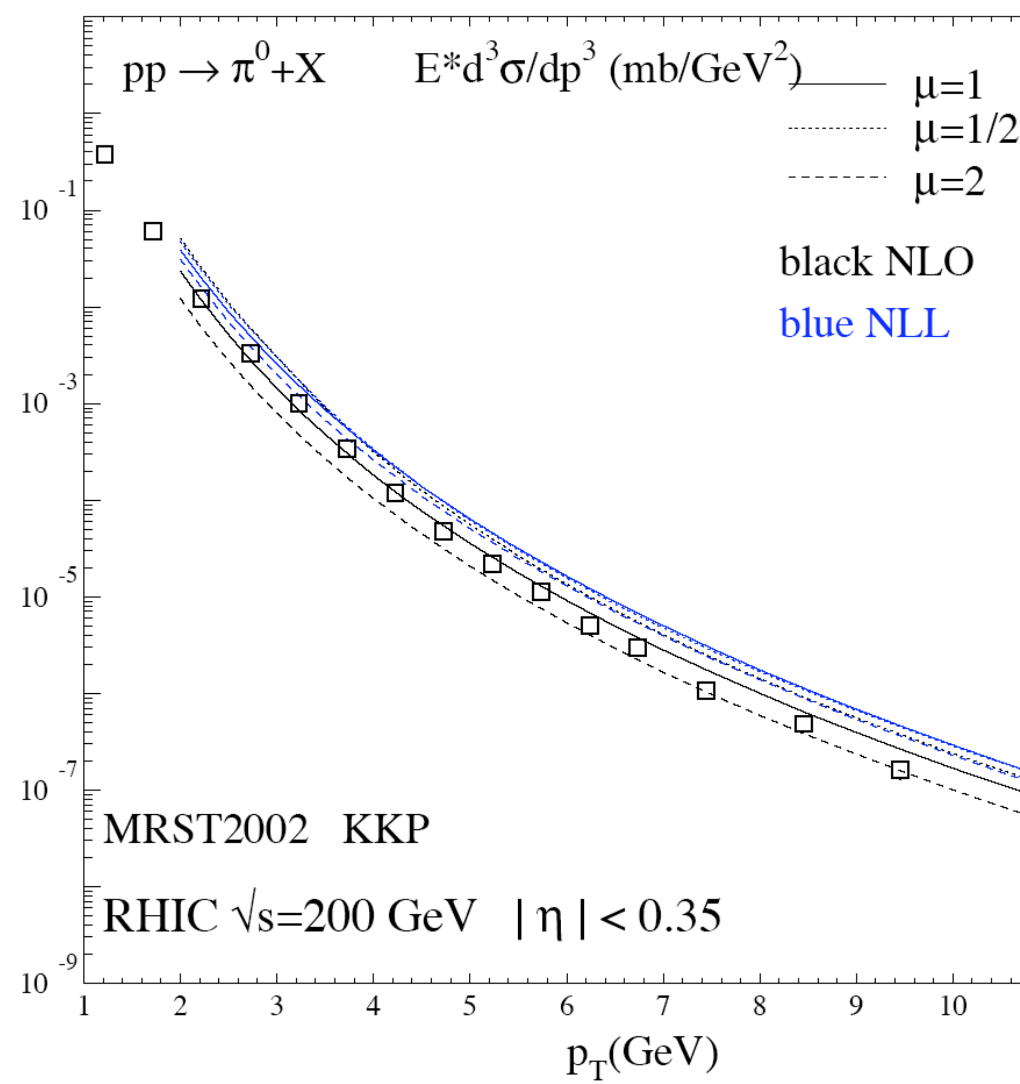
- **beginnings of analysis of power corrections implied by resummation :**
  energy dependence → smaller effects at colliders

- **a lot of work remains !**
  resummation: rapidity dependence
  subleading terms
  recoil effects at smaller p_T
  effects on spin asymmetries

  power corrections:
  full analysis, global fit ?



→ **subleading terms important at collider energies**



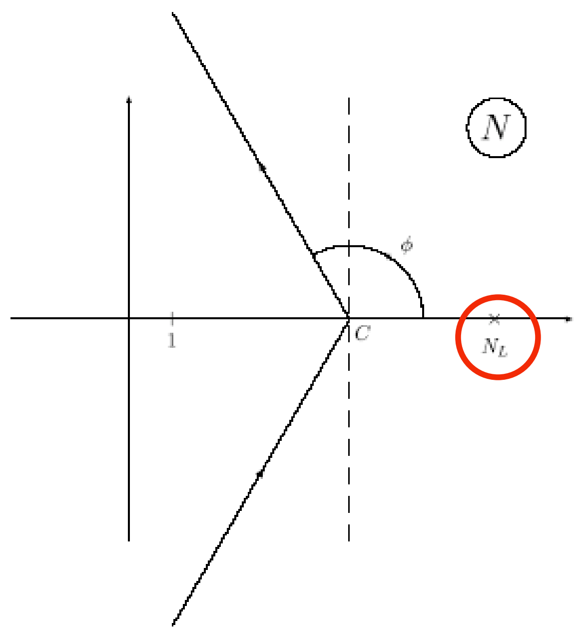
pp → π^0+X
E*d^3σ/dp^3 (mb/GeV^2)
μ=1
μ=1/2
μ=2
black NLO
blue NLL
MRST2002 KKP
RHIC √s=200 GeV |η| < 0.35
10^-1
10^-3
10^-5
10^-7
10^-9
1
2
3
4
5
6
7
8
9
10
11
p_T(GeV)

$$\exp\left[\frac{C_F}{\pi b_0^2 \alpha_s(Q^2)}\;[2\lambda + (1-2\lambda)\ln(1-2\lambda)] + \ldots\right]$$

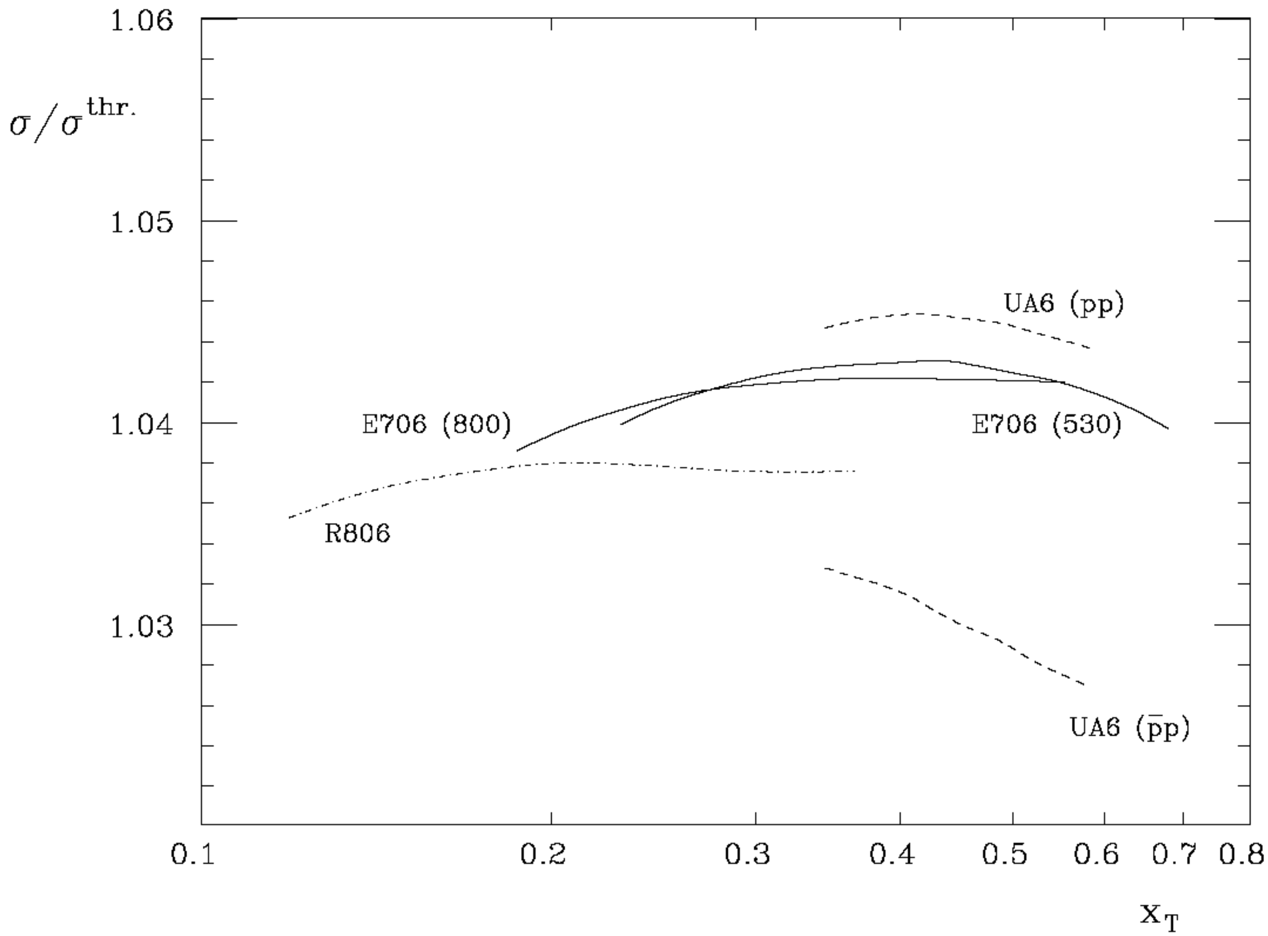
- **singular at** $\lambda = \frac{1}{2}$

$$\lambda = \alpha_s(Q^2) b_0 \ln(N)$$

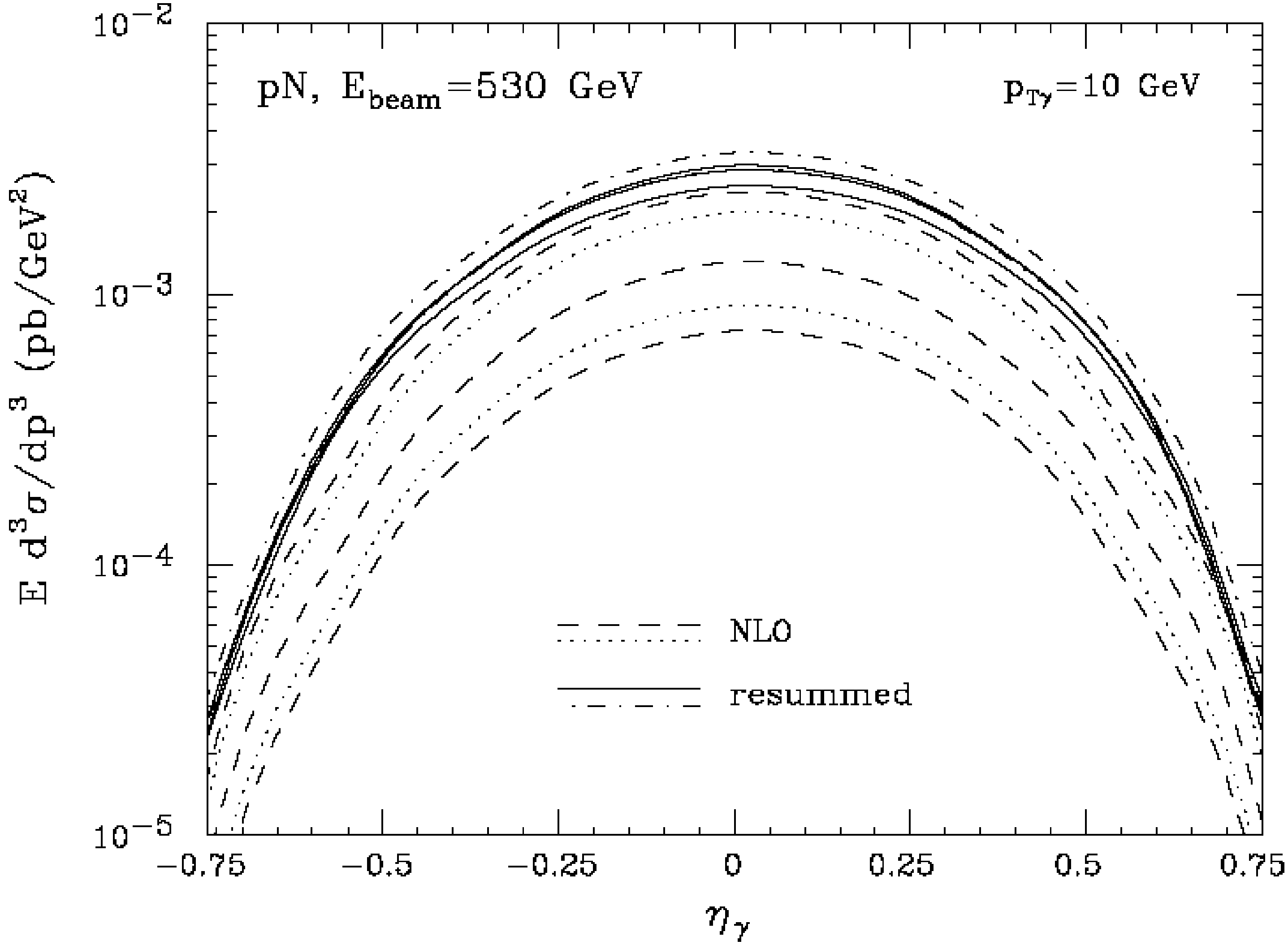
- **Mellin contour chosen as** Catani, Mangano, Nason, Trentadue



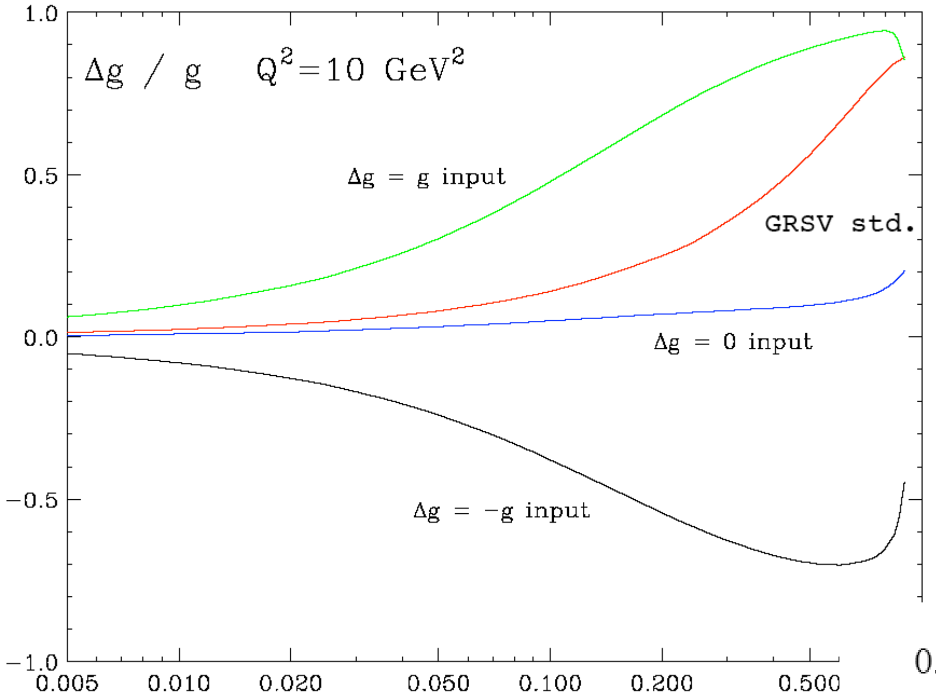
- **PT series defined in this way has no factorial growth**


1.06
$\sigma/\sigma^{thr.}$
1.05
1.04
1.03
UA6 (pp)
E706 (800)
E706 (530)
R806
UA6 ($\bar{p}$p)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
x_T

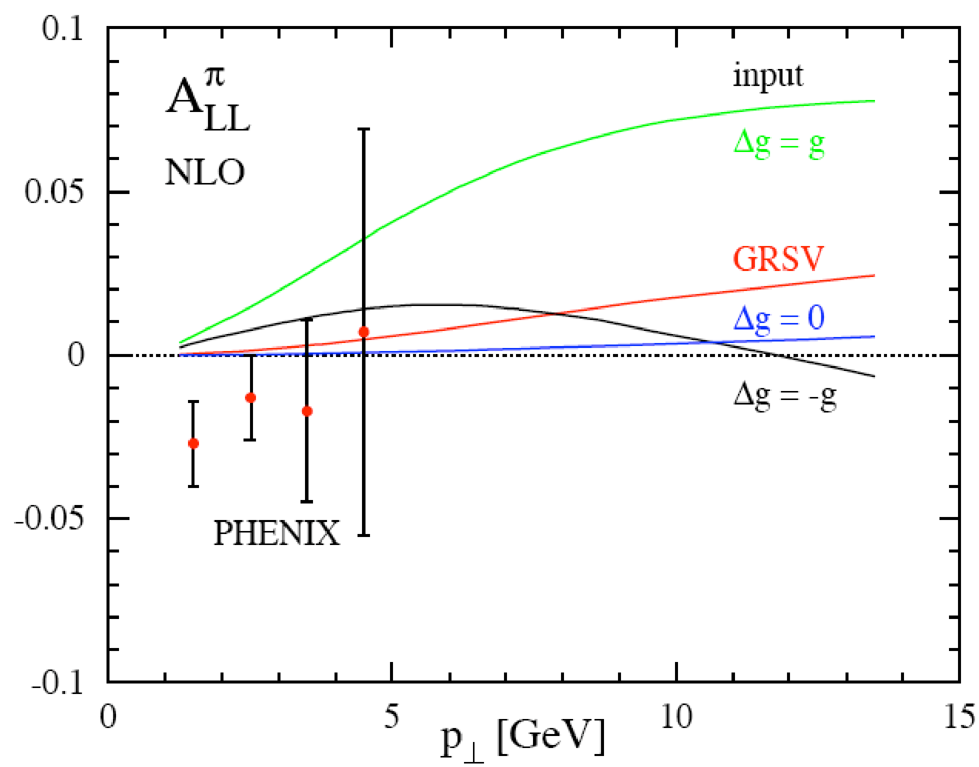
Sterman, WV


pN, E_{beam}=530 GeV
$p_{T\gamma}$=10 GeV
E $d^3\sigma/dp^3$ (pb/GeV^2)
η_γ
NLO
resummed
10^{-2}
10^{-3}
10^{-4}
10^{-5}
−0.75
−0.5
−0.25
0
0.25
0.5
0.75

Sterman, WV

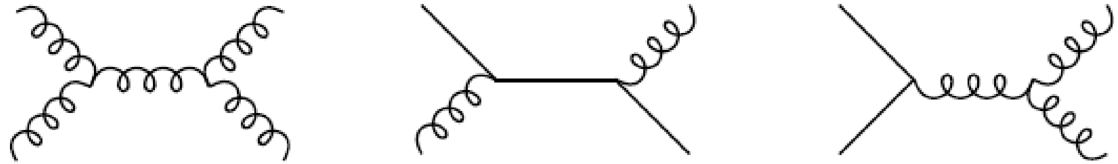


Glück,Reya,Stratmann,WV



Jäger, Stratmann, WV

recall : $gg,\ qg,\ qq$ scattering :



- therefore :

$$A_{LL} = (\Delta g(x))^2\ \mathcal{A} + \Delta g(x)\ \mathcal{B} + \mathcal{C}$$

($\mathcal{A} > 0$, $\mathcal{B} > 0$)

- a parabola – with a minimum that is negative, but tiny

$p_\perp = 1.5$ GeV : $A_{\rm LL}^{\pi} \approx -10^{-4}$

$p_\perp = 4.5$ GeV : $A_{\rm LL}^{\pi} \approx -10^{-3}$

- can be shown with more rigor Jäger,Kretzer,Stratmann,WV

Perturbative series :