Transversity and Drell–Yan K-Factors

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Motivation & Interest

The theoretical framework for describing transversity (at least at the basic level of partonic processes, quantum chromodynamics evolution, radiative effects, . . .) is now rather solid.

A number of experiments are on-line or under development: HERMES, COMPASS and the spin programme at RHIC.

Transversity is the last piece in the partonic jig-saw puzzle that makes up the hadronic picture.

Transverse-spin effects are notoriously surprising, *e.g.*, the large and unexpected (historically) single-spin asymmetries.

Models

Since they have opposite charge-conjugation properties, Δq and $\Delta_T q$ are not simply related.

Decomposing Δq as $\Delta q^{\mathrm{NS}} + \Delta q^{\mathrm{S}}$ one might imagine that

$$\Delta_T q \simeq \Delta q^{\mathrm{NS}}$$

In the non-relativistic quark model such is the case.

However, in a relativistic model the lower components spoil the identity: *e.g.*, the MIT bag gives (Jaffe and Ji, 1992):

$$\Delta q^{\mathrm{NS}} = c \int r^2 \mathrm{d}r \, (f^2 - \tfrac{1}{3} g^2)$$

$$\Delta_T q = c \int r^2 \mathrm{d}r \, (f^2 + \tfrac{1}{3} g^2)$$

where f, g (the upper, lower quark wave-function components) contribute differently due to the extra γ_0.

Leading Order DGLAP Evolution (Quarks)

The LO (non-singlet) DGLAP quark–quark splitting functions:

$$P_{qq}^{(0)}(x) = C_{\mathrm{F}} \left(\frac{1+x^2}{1-x} \right)_+$$

$$\Delta P_{qq}^{(0)}(x) = P_{qq}^{(0)}(x) \quad \text{helicity conservation}$$

$$\Delta_T P_{qq}^{(0)}(x) = P_{qq}^{(0)}(x) - C_{\mathrm{F}}(1-x)$$

N.B. For both $P_{qq}^{(0)}$ and $\Delta P_{qq}^{(0)}$ the first moments vanish (leading to well-known conservation laws and sum rules).

The same is not true for $\Delta_T P_{qq}^{(0)}$! So, no sum rules.

LO & NLO DGLAP Evolution

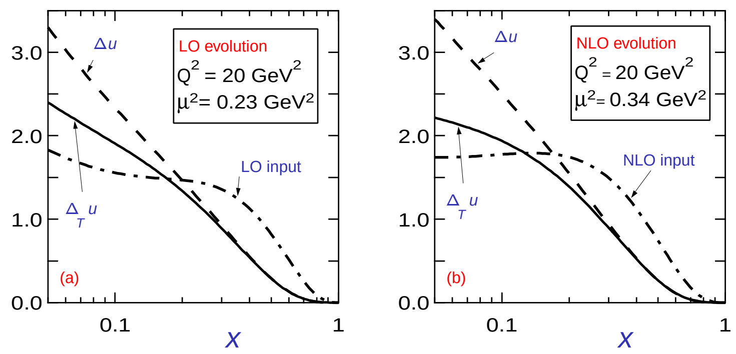


Figure: A comparison between the Q^2-evolution of $\Delta_T u(x, Q^2)$ and $\Delta u(x, Q^2)$ (a) at LO and (b) at NLO, from Hayashigaki *et al.* (1997).

Chirality Flip

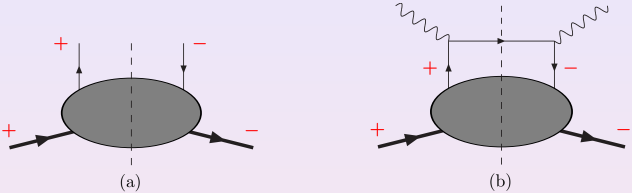


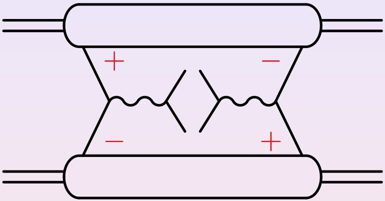
Figure: (a) Chirally-odd hadron–quark amplitude for h_1.
(b) Chirality-flip forbidden DIS handbag diagram.

All QCD and electroweak vertices conserve quark chirality.

N.B. Charged-current interactions exclude transversity since only a single chirality interacts.

N.B. Chirality flip is not a problem if the quarks can connect to different hadrons, *e.g.*, as in Drell–Yan.

Drell–Yan and Hikasa's Theorem



A caveat to accessing transversity in DY is a theorem due to Hikasa (1986):

chiral symmetry requires that the azimuth of the lepton pair remain unintegrated.

No simple proof exists (it has to do with γ-matrix properties).

NLO Drell–Yan and K Factors

Quark densities are normally defined in DIS, where the parton picture is formulated and model calculations performed.

When translated to DY, large radiative K factors are generated $\sim O(\pi\alpha_s)$, enhancing the cross-sections (Altarelli *et al*., 1979).

At RHIC energies the correction is roughly 30% while at EMC/SMC energies it becomes nearly 100%.

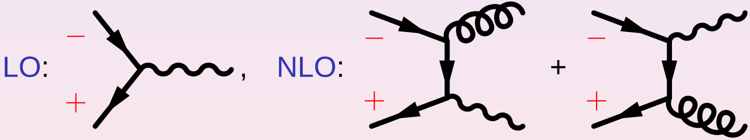
N.B. Spin asymmetries are ratios of differences and sums of cross-sections for different spin-alignment combinations.

Thus, any strong polarisation dependence in the K factors could lead to dramatic variations in the asymmetries—with respect say to model predictions.

NLO and K Factors for Helicity

For the $q\bar{q}$ annihilation contribution and longitudinal hadron polarisation, this turns out not to be the case (PGR, 1983).

This can be understood (at least partially) from the helicity-conserving nature of the vector interaction.



Thus, only one helicity combination contributes (up to NLO).

(For gluon–quark scattering the situation is not quite so clean.)

NLO and K Factors for Transversity

The case of transversity is somewhat peculiar: no standard DIS definition exists, as discussed earlier.

Neither is it obvious here that quark helicity-conservation should still save the day.

The pure DY NLO coefficient functions are known in various schemes (Vogelsang *et al*., 1993; Contogouris *et al*., 1994).

Of course, they are, indeed, scheme dependent.

Surprisingly, a new term $\propto \frac{z \ln^2 z}{1-z}$ appears, which is not found for either spin-averaged or helicity-dependent DY.

NLO and K Factors for Transversity

To study the K-factor problem, we need a DIS-like process to which transversity may contribute. Thus, we require a DIS helicity-flip mechanism—this could be provided by:

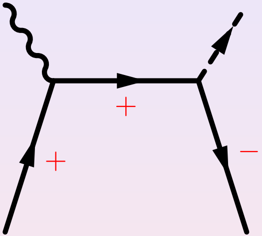
- a quark mass (*i.e.*, in a propagator);
- a scalar vertex (*e.g.*, a Higgs coupling).

Although a quark mass does, in principle, do what is required, the contribution cancels owing to the equations of motion and gauge invariance (see, *e.g.*, Anselmino *et al.*, 1995).

A (single) Higgs-like vertex, interfering with the normal photon vertex does allow a chiral-odd contribution to DIS (Ioffe *et al.*, 1995, from a suggestion by Jaffe).

Indeed, such a *gedanken* process may be used to calculate the anomalous dimensions, but care is needed.

Interpolating Currents



$$= J_V \cdot J_S = \sum_n C_n(\xi)\, \mathcal{O}_n(0)$$

A first attempt at calculating γ with this method led to an apparent contradiction—corrected by Blümlein (2001):

The vector current J_V is conserved and thus has $\gamma_V = 0$,

However, the scalar current J_S is not conserved and $\gamma_S \neq 0$.

Renormalisation Group

The product of two currents may be expanded as

$$J_V(\xi) \cdot J_S(0) = \sum_n C(n;\xi)\ \mathcal{O}(n;0)$$

The RGE's for the Wilson coefficients $C(n;\xi)$ are

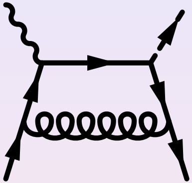
$$\left[\mathcal{D} + \gamma_{J_V}(g) + \gamma_{J_S}(g) - \gamma_{\mathcal{O}}(n;g)\right] C(n;\xi) = 0$$

Thus, the "Compton" amplitude correction has coefficient

$$\gamma_C(n;g) = \gamma_{J_V}(g) + \gamma_{J_S}(g) - \gamma_{\mathcal{O}}(n;g)$$

Therefore, $\gamma_{\mathcal{O}} \neq \gamma_C$!

DIS Higgs–Photon Interference



Since the scalar current is not conserved, there is an extra UV contribution from the scalar vertex, which is factorised into the Higgs coupling constant (or equivalently, the running quark mass).

$$C^f_{q,\mathrm{DY}} - 2C^f_{q,\mathrm{DIS}} = \frac{\alpha_s}{2\pi} C_\mathrm{F} \left[\frac{3}{(1-z)_+} + 2\left(1+z^2\right)\left(\frac{\ln(1-z)}{1-z}\right)_+ - 6 - 4z + \left(C_\mathrm{F}\pi^2 + 1\right)\delta(1-z) \right]$$

$$C^g_{q,\mathrm{DY}} - 2C^g_{q,\mathrm{DIS}} = C^f_{q,\mathrm{DY}} - 2C^f_{q,\mathrm{DIS}} + \frac{\alpha_s}{2\pi} C_\mathrm{F}\, 2(1+z)$$

$$C^h_{q,\mathrm{DY}} - 2C^h_{q,\mathrm{DIS}} = \frac{\alpha_s}{2\pi} C_\mathrm{F} \left[\frac{3}{(1-z)_+} + 4z\left(\frac{\ln(1-z)}{1-z}\right)_+ - 6z\,\frac{\ln^2 z}{1-z} + 1 - 4z + \left(C_\mathrm{F}\pi^2 + 1\right)\delta(1-z) \right]$$

The origins of these differences may be traced to the different phase-space restrictions in the transversity case.

DIS Higgs–Photon Interference

One could, of course, argue that it is the Higgs-like vertex that spoils the cancellation of K-factors in the transversity case.

However, one can also construct Drell–Yan processes in which the lepton pair is produced via an intermediate Higgs state. Here there is no funny phase space problem as the presence of scalar (chirality-flip) vertices avoids Hikasa's theorem and the final lepton-pair azimuth may be integrated out.

Likewise, a purely Higgs-exchange DIS process exists.

In these cases the large K-factors are again "well-behaved".

Moral: model calculations might not fair too well at first sight if not suitably corrected for the transition from DIS to DY.

DIS–DY Asymmetry

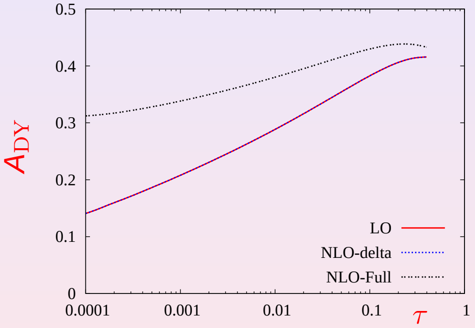


Figure: Transversity asymmetry (valence only) for Drell–Yan ($\tau = Q^2/s$, $s = 4 \cdot 10^4\,\mathrm{GeV}^2$, kinematic limits $\tau < x_1, x_2 < 1$).

DIS–DY Coefficient Differences

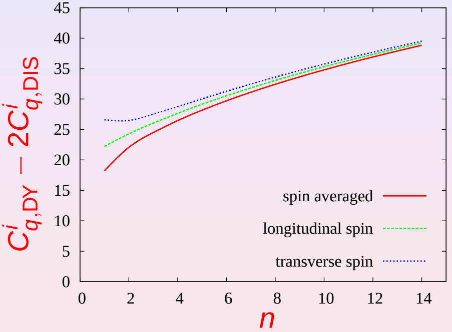


Figure: Spin-averaged, helicity and transversity coefficient differences ($C^i_{q,\mathrm{DY}} - 2C^i_{q,\mathrm{DIS}}$ for $i = f, g, h$) in Mellin moment space.

Summary and Conclusions

Transversity is equally important a part of nucleon structure as the other two leading-twist parton densities (spin-averaged and helicity-weighted). A complete description of the nucleon must include this aspect of parton dynamics.

On the theory side, there is a complete and standard QCD (including NLO) picture (essentially following in the well-trodden footsteps of the previous two structures), but only in DY (or rather more exotic processes).

On the experimental side, as yet, we have no real knowledge. However, the future is promising and before too long we should start to harvest interesting results.

Summary and Conclusions

The phenomenology, while not dissimilar to the other two leading-twist structures, has some interesting peculiarities:

Transversity is not measurable in standard fully inclusive DIS.

In DY Hikasa's theorem forces us to maintain the lepton azimuth unintegrated.

This leads to a new term in the NLO corrections, which substantially alters the effective NLO K-factor.

Thus, comparison with model predictions and even the Soffer bound may be misleading. Moreover, any more exotic process may equally suffer large NLO asymmetry corrections.

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