Understanding transversity: present and future

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Outline

1. What we know about transversity
    - Partonic meaning
    - QCD evolution
    - Non-perturbative results
    - k_T-dependent distributions
2. Probing transversity: present and future
    - Single-spin asymmetries (SSA): k_T dependence, Collins and Sivers effects, SIDIS and hadroproduction data and their interpretation
    - Double-spin asymmetries (DSA): collinear factorization, QCD predictions
3. Summary and conclusions

Transversity in elementary terms

Transversity $\equiv$ Transverse Polarization $(\Pi = \frac{1}{2}\gamma_5\gamma_\perp = \frac{1}{2}\gamma_0\Sigma_\perp)$

Transversity states are linear combinations of helicity states: $|\uparrow\downarrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle \pm i\,|-\rangle)$

Transversely polarized quarks ($\uparrow\downarrow$) in a transversely polarized proton ($\Uparrow$):

Transversity distribution $\Delta_T f^q(x) = f^q_\uparrow(x) - f^q_\downarrow(x)$ [also called $h_1^q(x)$]

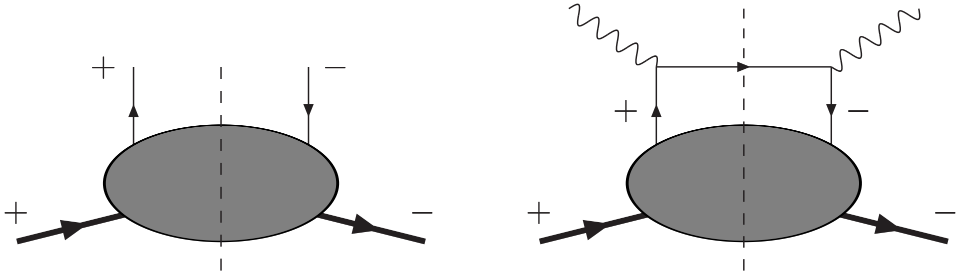
[analogous to the helicity distribution $\Delta f^q(x) = f^q_+(x) - f^q_-(x)$, also called $g_1^q(x)$]

Properties of h_1^q:

- leading twist (unsuppressed)
- chirally odd (non diagonal in helicity basis)
- non-singlet evolution (no gluonic h_1)

Transversity is not probed in inclusive DIS: no handbag diagram

chirally
odd



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To observe transversity, helicity must flipped twice, so one needs at least two hadrons (hadron-hadron scattering or semi-inclusive DIS)

There is no gluonic transversity distribution for spin-$\frac{1}{2}$ hadrons: helicity-flip gluon-nucleon amplitudes cannot exist (they would imply $\Delta\lambda = \pm 2$)

A bit of history

Transversity is by now 25 years old, but looks younger (it was forgotten for 10 years)

- Introduced by Ralston and Soper (1979) in transversely polarized Drell-Yan
- Rediscovered and studied by Artru & Mekhfi (1990), Jaffe & Ji (1991), Cortes, Pire & Ralston (1992)
- See also Kodaira et al. (1979), Bukhvostov, Kuraev & Lipatov (1981), Efremov & Teryaev (1984), Ratcliffe (1986): quark mass contribution to the twist-3 part of g_2
- Theoretical investigation of transversity and related observables: various authors (1990-2000). First experimental results: E704 (1991), SMC (1999), HERMES (2000)

[For an overview, see VB, Drago & Ratcliffe, Phys. Rep. 359 (2002) 1]

Highlights of 2002-2004:

- Better understanding of k_T-dependent distributions (Wilson link structures, Sivers function, time-reversal modified universality of parton densities, etc.)
- NLO QCD analyses of transversely polarized collinear hard processes ($p^\uparrow p^\uparrow \to \gamma X$, $ep^\uparrow \to e'\Lambda^\uparrow X$, etc.)
- More detailed investigations of transversely polarized lepto- and hadro-production (k_T effects, two-particle final states, higher twists, etc.)
- New processes proposed and studied ($pp^\uparrow \to j_1 j_2 X$, $pp^\uparrow \to h_1 h_2 X$, $pp^\uparrow \to DX$, $p^\uparrow \bar{p}^\uparrow \to \mu^+\mu^-(J/\psi)X$, etc.)
- Experiments: data on SSA's in $ep^\uparrow$ (HERMES, COMPASS), new results on SSA in $pp^\uparrow$ at higher energy (STAR)

Notations and terminology

$\Delta_T f_q(x) \equiv h_1^q(x)$ transversity distribution function

$\Delta^N f_{q/p^\uparrow}(x, k_T^2) \equiv -\frac{2\,|\boldsymbol{k}_T|}{M}\, f_{1T}^{\perp q}(x, k_T^2)$ Sivers distribution function

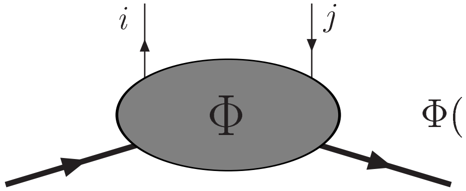
$\Delta^N D_{h/q^\uparrow}(z, P_{hT}^2) \equiv \frac{2\,|\boldsymbol{P}_{hT}|}{zM_h}\, H_1^{\perp q}(z, P_{hT}^2)$ Collins fragmentation function

Mulders & Tangerman, NP B461 (1996) 197

VB, Drago & Ratcliffe, Phys. Rep. 359 (2002) 1

Bacchetta, D'Alesio, Diehl & Miller, Trento conventions, hep-ph/0410050

The three leading-twist quark distribution functions



$$\Phi(x) \sim f_1(x) \not{P} + \lambda_N g_1(x) \gamma_5 \not{P} + h_1(x) \not{P} \gamma_5 \not{S}_T$$

number $$f_1(x) = \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-} \, \langle P,S|\overline{\psi}(0)\gamma^+\psi(\xi)|P,S\rangle\Big|_{\xi^+=\vec{\xi}_\perp=0}$$

helicity $$g_1(x) = \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-} \, \langle P,S|\overline{\psi}(0)\gamma^+\gamma_5\psi(\xi)|P,S\rangle\Big|_{\xi^+=\vec{\xi}_\perp=0}$$

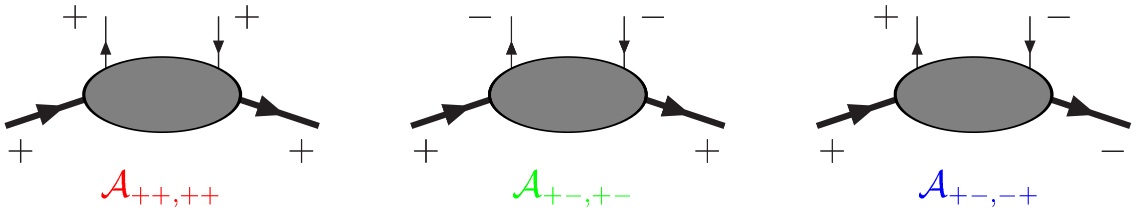
transversity $$h_1(x) = \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-} \, \langle P,S|\overline{\psi}(0)\gamma^+\gamma_\perp\gamma_5\psi(\xi)|P,S\rangle\Big|_{\xi^+=\vec{\xi}_\perp=0}$$

[Wilson line to ensure gauge invariance: in light-cone gauge, $A^+ = 0$, it reduces to $\mathbb{1}$]

tensor charge: first moment of $h_1 - \overline{h}_1$

$$\langle P,S|\overline{\psi}(0)\mathrm{i}\sigma^{\mu\nu}\gamma_5\psi(0)|P,S\rangle = 2\,\delta q\, S^{[\mu}P^{\nu]}\,, \qquad \delta q = \int_0^1 \mathrm{d}x\,[h_1^q(x) - \overline{h}_1^q(x)]$$

Quark–nucleon helicity amplitudes



$$f_1(x) = f_+(x) + f_-(x) \sim \mathrm{Im}\,(\mathcal{A}_{++,++} + \mathcal{A}_{+-,+-}) = \sum_X (a^*_{++}a_{++} + a^*_{+-}a_{+-})$$

$$g_1(x) = f_+(x) - f_-(x) \sim \mathrm{Im}\,(\mathcal{A}_{++,++} - \mathcal{A}_{+-,+-}) = \sum_X (a^*_{++}a_{++} - a^*_{+-}a_{+-})$$

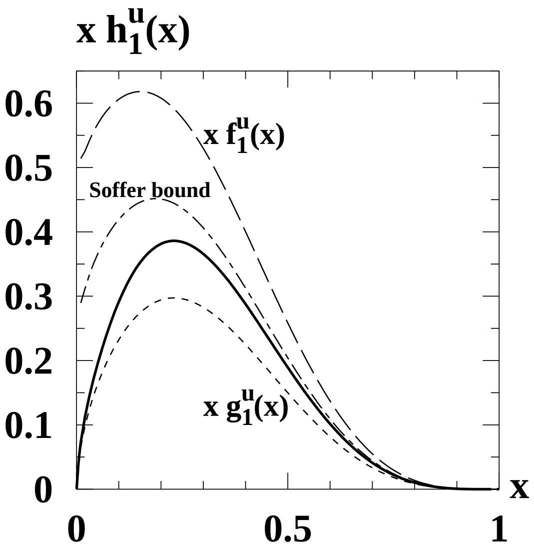
$$h_1(x) = f_\uparrow(x) - f_\downarrow(x) \sim \mathrm{Im}\,\mathcal{A}_{+-,-+} = \sum_X a^*_{--}a_{++}$$

From $\sum_X |a_{++} \pm a_{--}|^2 \geq 0$ follows the Soffer inequality [Soffer 1995]:

$$|h_1(x)| \leq \frac{1}{2}\,[f_1(x) + g_1(x)] = f_+(x)$$

This is an important constraint relating the three leading-twist distributions:

- Must be checked or implemented in modeling h_1



[Chiral quark soliton model, Efremov, Goeke & Schweitzer]

- Provides upper bounds on some transversity observables

QCD evolution of transversity

Purely non-singlet evolution (for spin $\frac{1}{2}$ hadrons): no mixing with gluons

LO (Artru & Mekhfi):

$$\Delta_T \gamma_{qq}^{(0)} = \Delta \gamma_{qq}^{(0)} - \frac{C_F}{n(n+1)}\,, \qquad \Delta_T P_{qq}^{(0)} = \Delta P_{qq}^{(0)} - C_F(1-z)$$

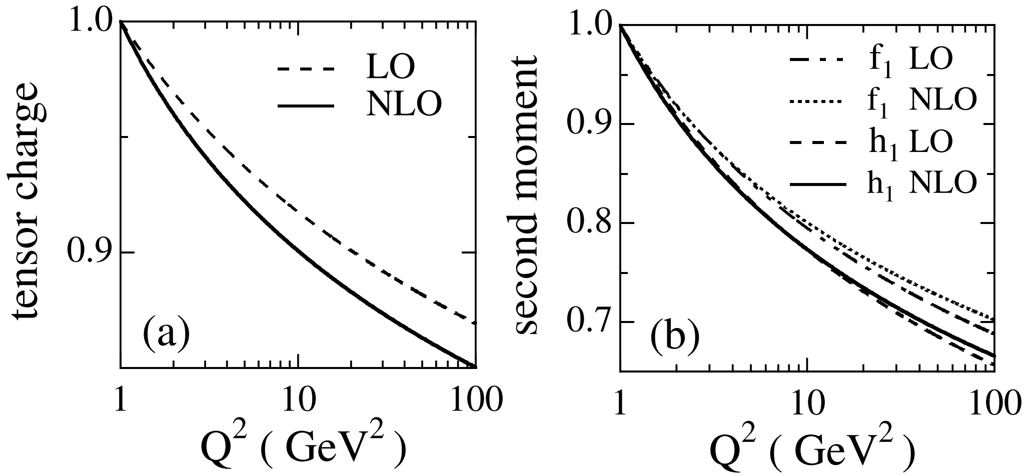
First moment (tensor charge), $\Delta_T \gamma_{qq}^{(0)} = -\frac{2}{3}$:

$$\delta q(Q^2) = \left[\frac{\alpha_S(Q^2)}{\alpha_S(Q_0)^2}\right]^{\frac{4}{27}} \delta q(Q_0^2) \quad \text{decreases slowly}$$

NLO (Hayashigaki, Kanazawa & Koike; Kumano & Miyama; Vogelsang):

$$\delta q(Q^2) = \left[\frac{\alpha_S(Q^2)}{\alpha_S(Q_0)^2}\right]^{\frac{4}{27}} \left[1 - \frac{337}{486\,\pi}\left(\alpha_S(Q_0^2) - \alpha_S(Q^2)\right)\right] \delta q(Q_0^2)$$

[Hayashigaki, Kanazawa & Koike 1997]



Angular momentum sum rule for transversity [Bakker, Leader & Trueman 2004]:

$$\frac{1}{2} = \frac{1}{2} \sum_{a=q,\bar{q}} \int \mathrm{d}x \, h_1^a(x) + \sum_{a=q,\bar{q},g} \langle L_T \rangle^a$$

Since transversity decreases with increasing Q^2, the orbital angular momentum must increase (assuming an initial zero value) [Ratcliffe]

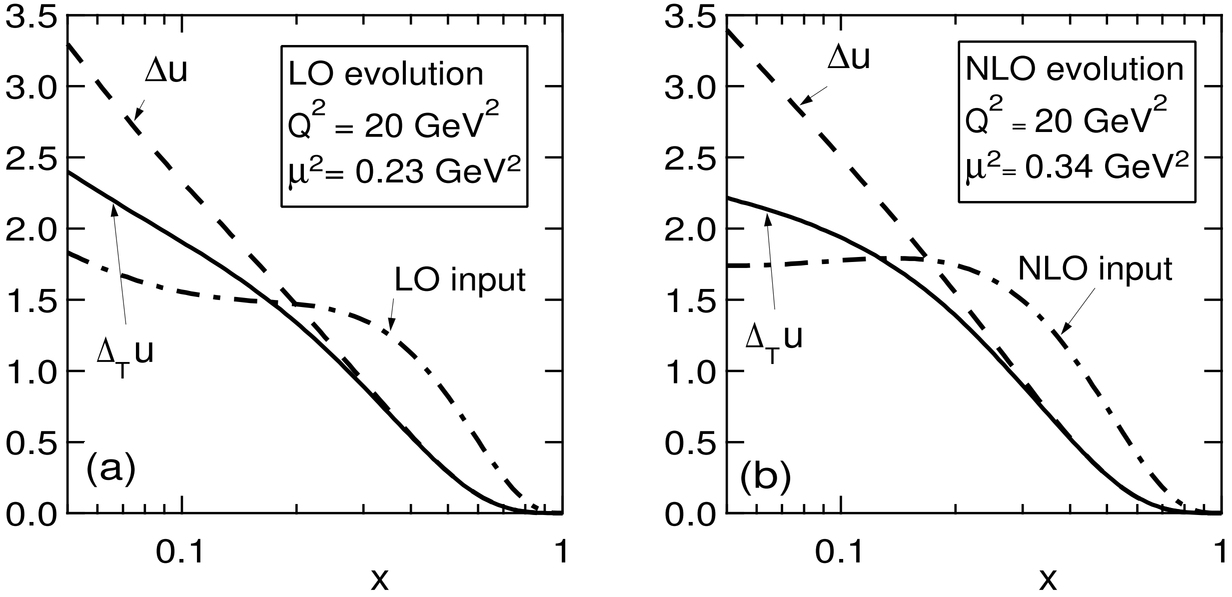
For spin-1 hadrons, there is also the gluonic transversity h_1^g term, which increases with increasing Q^2

It would be extremely interesting to study this sum rule in pQCD

Evolution of h_1 rather different from that of g_1, especially at small x [VB 1997]

$$\text{splitting functions:} \quad \Delta_T P_{qq} \underset{x \to 0}{\sim} x\,, \qquad \Delta P_{qq} \underset{x \to 0}{\sim} \text{const.}$$

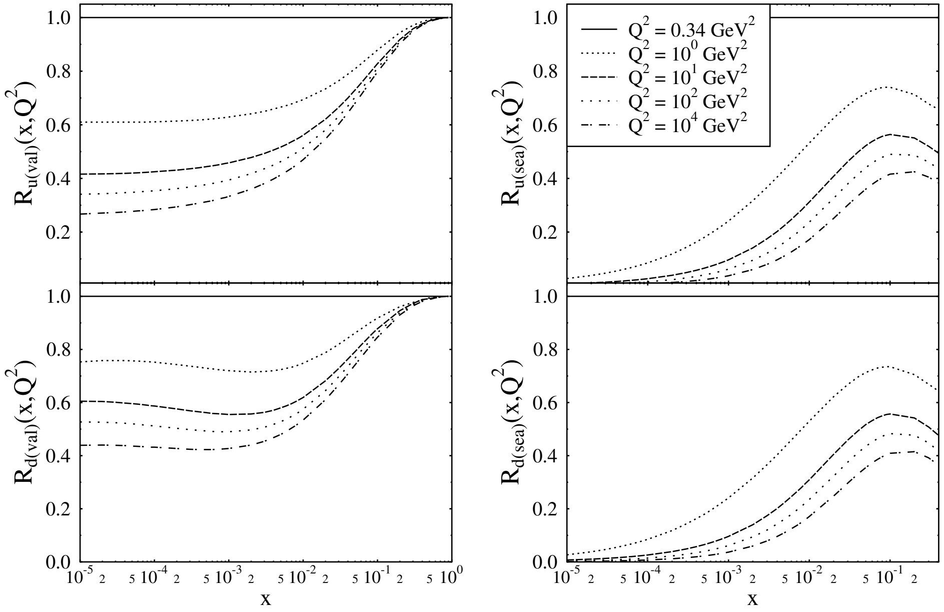
At small x, h_1 is suppressed compared to g_1



[Hayashigaki, Kanazawa & Koike 1997], $h_1 = g_1$ at the input (GRV) scale

Soffer inequality in QCD: $|h_1(x,Q^2)| \leq \frac{1}{2}[f_1(x,Q^2) + g_1(x,Q^2)]$

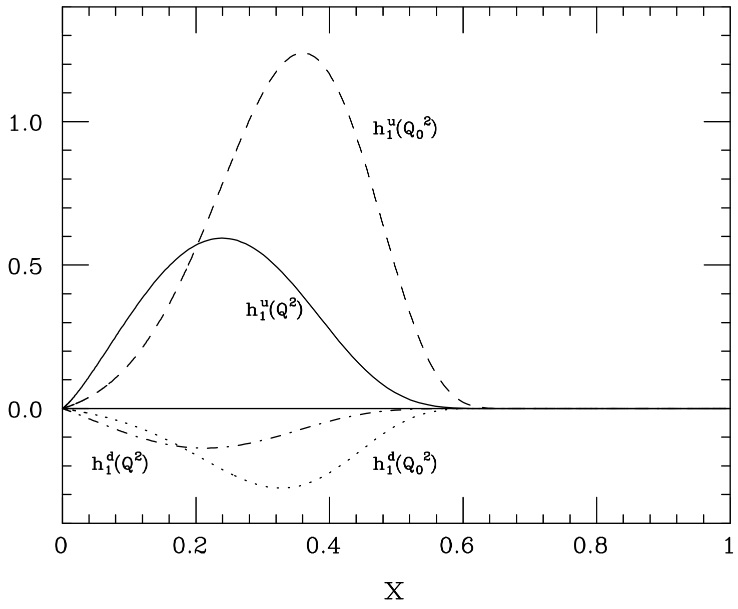
- Preserved at LO [VB 1997]
- At NLO, parton distributions are well defined only within a renormalization scheme
- Schemes can be found which preserve the Soffer inequality at NLO [Martin et al. 1998, Bourrely, Soffer and Teryaev 1998]



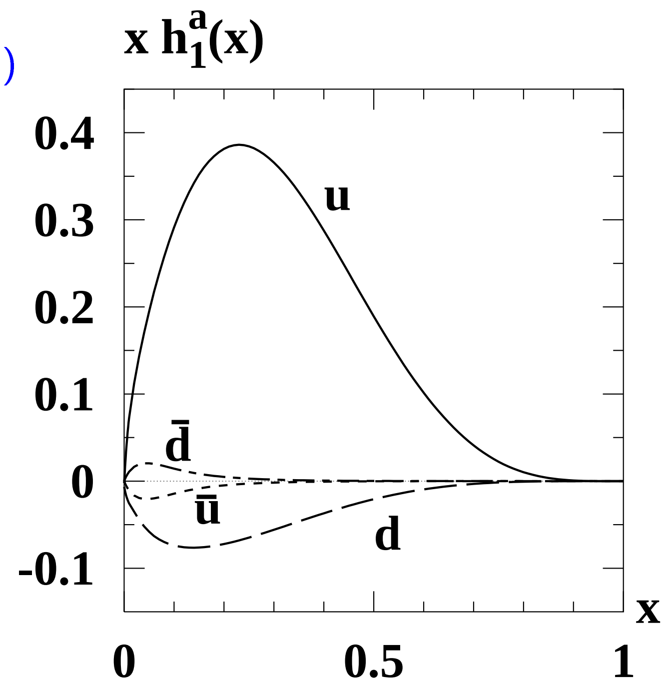
$R = 2\,|h_1|/(f_1 + g_1)$ [Martin, Schäfer, Stratmann & Vogelsang 1998]

Model calculations of h_1 (two examples)

Chiral chromodielectric model (VB, Calarco & Drago)

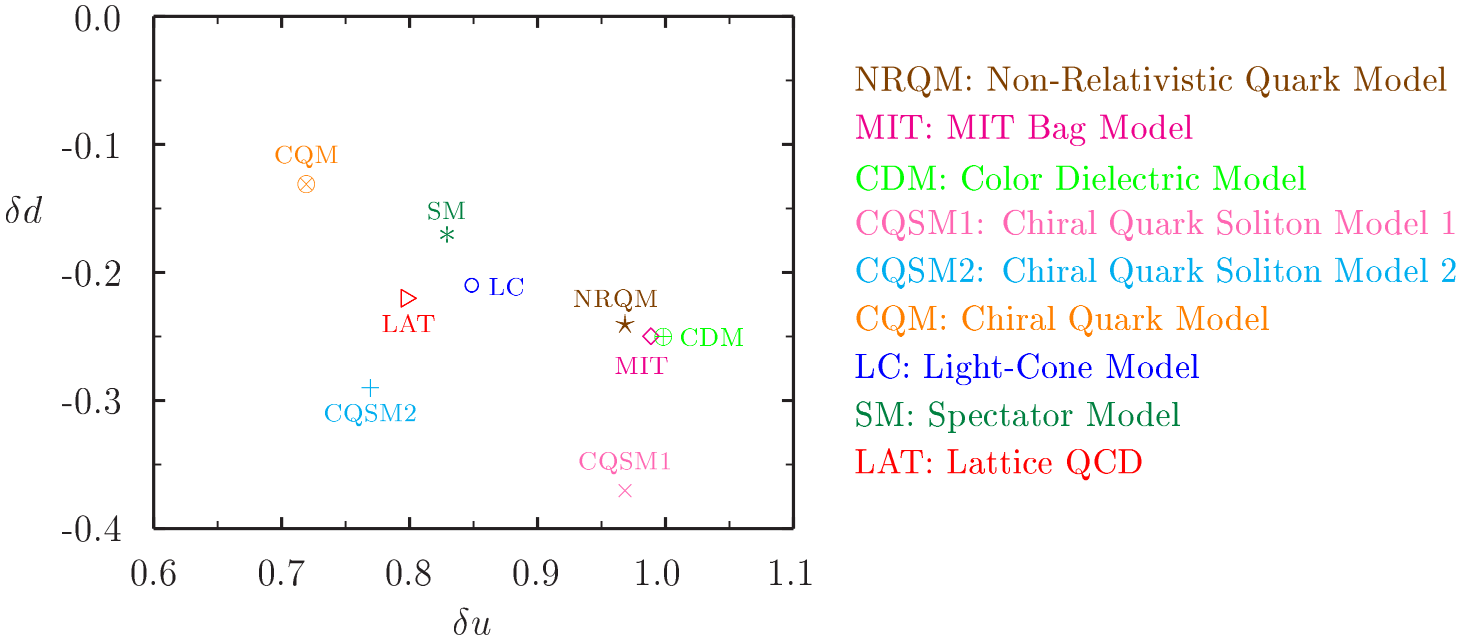


Chiral quark soliton model (Schweitzer et al.)



- $h_1 \simeq g_1$ at a very low scale (in NRQM, $h_1 = g_1$; in relativistic models the difference is due to lower components of wf's)
- Models differ for the sign of $\bar{u}$ (different procedures to compute antiquark distributions [VB, Calarco, Drago & Simani 2003])

Tensor charges in various models



NRQM: Non-Relativistic Quark Model
MIT: MIT Bag Model
CDM: Color Dielectric Model
CQSM1: Chiral Quark Soliton Model 1
CQSM2: Chiral Quark Soliton Model 2
CQM: Chiral Quark Model
LC: Light-Cone Model
SM: Spectator Model
LAT: Lattice QCD

$$\delta u \sim 0.7 - 1.0, \quad \delta d \sim -(0.1 - 0.4) \quad \text{at} \quad Q^2 = 10 \text{ GeV}^2$$

The relative weight of δu and δd is important for understanding data on single-spin asymmetries in $\pi^{\pm}, \pi^0$ leptoproduction

Transverse motion of quarks

[Mulders et al., Kotzinian, ...]

k_T-dependent distributions are relevant for single-spin asymmetries

Various possible correlations between $\boldsymbol{k}_T$, $\boldsymbol{S}_T$ and $\boldsymbol{S}_{qT}$

Transversely polarized quarks inside a transversely polarized proton:

$$\begin{aligned} \mathcal{P}_{q^\uparrow/p^\uparrow}(x,\boldsymbol{k}_T) &= \frac{1}{2}\Big\{ f_1(x,\boldsymbol{k}_T^2) + (\boldsymbol{S}_T\cdot\boldsymbol{S}_{qT})\, h_1(x,\boldsymbol{k}_T^2) \\ &\quad - \frac{1}{M^2}\left[(\boldsymbol{k}_T\cdot\boldsymbol{S}_{qT})(\boldsymbol{k}_T\cdot\boldsymbol{S}_T) + \frac{1}{2}\,\boldsymbol{k}_T^2(\boldsymbol{S}_T\cdot\boldsymbol{S}_{qT})\right] h_{1T}^\perp(x,\boldsymbol{k}_T^2)\Big\} \end{aligned}$$

$$\begin{aligned} \mathcal{P}_{q^\uparrow/p^\uparrow}(x,\boldsymbol{k}_T) - \mathcal{P}_{q^\downarrow/p^\uparrow}(x,\boldsymbol{k}_T) &= (\boldsymbol{S}_T\cdot\boldsymbol{S}_{qT})\, h_1(x,\boldsymbol{k}_T^2) \\ &\quad - \frac{1}{M^2}\left[(\boldsymbol{k}_T\cdot\boldsymbol{S}_{qT})(\boldsymbol{k}_T\cdot\boldsymbol{S}_T) + \frac{1}{2}\,\boldsymbol{k}_T^2(\boldsymbol{S}_T\cdot\boldsymbol{S}_{qT})\right] h_{1T}^\perp(x,\boldsymbol{k}_T^2) \end{aligned}$$

Both h_1 and $h_{1T}^\perp$ contribute to SSA in $ep^\uparrow$ (via Collins effect), but with different angular distributions, $\sin(\phi_h+\phi_S)$ and $\sin(3\phi_h-\phi_S)$

Unpolarized quarks inside a transversely polarized proton:

$$\mathcal{P}_{q/p^\uparrow}(x, \boldsymbol{k}_T) = f_1(x, \boldsymbol{k}_T^2) + \frac{(\boldsymbol{k}_T \times \hat{\boldsymbol{P}}) \cdot \boldsymbol{S}_T}{M} f_{1T}^\perp(x, \boldsymbol{k}_T^2)$$

$$\mathcal{P}_{q/p^\uparrow}(x, \boldsymbol{k}_T) - \mathcal{P}_{q/p^\uparrow}(x, -\boldsymbol{k}_T) = \frac{(\boldsymbol{k}_T \times \hat{\boldsymbol{P}}) \cdot \boldsymbol{S}_T}{M} f_{1T}^\perp(x, \boldsymbol{k}_T^2)$$

$f_{1T}^\perp$ is the Sivers distribution function: azimuthal asymmetry of unpolarized quarks inside a transversely polarized proton

Transversely polarized quarks inside an unpolarized proton:

$$\mathcal{P}_{q^\uparrow/p}(x, \boldsymbol{k}_T) = \frac{1}{2}\left[f_1(x, \boldsymbol{k}_T^2) + \frac{(\boldsymbol{k}_T \times \hat{\boldsymbol{P}}) \cdot \boldsymbol{S}_{qT}}{M} h_1^\perp(x, \boldsymbol{k}_T^2) \right]$$

$$\mathcal{P}_{q^\uparrow/p}(x, \boldsymbol{k}_T) - \mathcal{P}_{q^\downarrow/p}(x, \boldsymbol{k}_T) = \frac{(\boldsymbol{k}_T \times \hat{\boldsymbol{P}}) \cdot \boldsymbol{S}_{qT}}{M} h_1^\perp(x, \boldsymbol{k}_T^2)$$

$h_1^\perp$ is the Boer-Mulders distribution function: spin asymmetry of transversely polarized quarks inside an unpolarized proton

“T-odd” distributions $f_{1T}^{\perp}$ and $h_1^{\perp}$

$f_{1T}^{\perp}$ and $h_1^{\perp}$ measure T-odd correlations: $(\hat{\boldsymbol{P}} \times \boldsymbol{k}_T) \cdot \boldsymbol{S}_T$ and $(\hat{\boldsymbol{P}} \times \boldsymbol{k}_T) \cdot \boldsymbol{S}_{qT}$

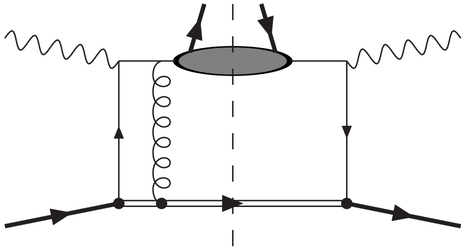
Operator definition of the Sivers function:

$$f_{1T}^{\perp}(x, \boldsymbol{k}_T^2) \sim \int d\xi^- \int d\boldsymbol{\xi}_T \, e^{ixP^+\xi^- - i\mathbf{k}_T \cdot \xi_T} \langle P, S_T | \overline{\psi}(\xi) \gamma^+ W(0,\xi) \psi(0) | P, S_T \rangle$$

If we set the Wilson link W to $\mathbb{1}$, time reversal implies [Collins 1993]

$$f_{1T}^{\perp}(x, \boldsymbol{k}_T^2) = -f_{1T}^{\perp}(x, \boldsymbol{k}_T^2) \qquad \Rightarrow \qquad \text{Sivers function vanishes}$$

But an explicit calculation [Brodsky, Hwang & Schmidt 2002] shows that



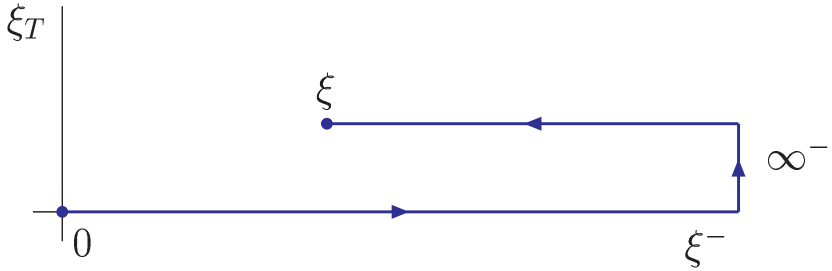
$$\Rightarrow \qquad f_{1T}^{\perp} \neq 0$$

Gluon exchange between struck quark and spectator generates Sivers asymmetry.
Puzzle solved by carefully considering Wilson lines [Collins 2002]

Wilson-line structure of k_T-dependent distributions

$$\mathcal{P}_{q/p}(x, \boldsymbol{k}_T) = \int \frac{\mathrm{d}\xi^-}{4\pi} \int \frac{\mathrm{d}\boldsymbol{\xi}_T}{(2\pi)^2}\, \mathrm{e}^{\mathrm{i}xP^+\xi^- - \mathrm{i}\mathbf{k}_T\cdot\xi_T} \langle P, S|\overline{\psi}(\xi)\gamma^+ W(0,\xi)\psi(0)|P, S\rangle$$

Wilson line $W_\gamma = \mathrm{P}\ \exp\left[-ig\int_\gamma A(x)\cdot \mathrm{d}x\right]$: path γ depends on the process



SIDIS : $W(0,\xi) = W(0^-, 0_T; \infty^-, 0_T)W(\infty^-, 0_T; \infty^-, \xi_T)W(\infty^-, \xi_T; \xi^-, \xi_T)$

The gauge potential at ∞^- does not vanish in light-cone gauge [Belitsky, Ji & Yuan 2003]

Time reversal implies

$$\mathcal{P}_{q/p^\uparrow}(x,\boldsymbol{k}_T)\Big|_{\text{future pointing } W} = \mathcal{P}_{q/p^\downarrow}(x,\boldsymbol{k}_T)\Big|_{\text{past pointing } W}$$

SIDIS: Sivers asymmetry from interaction between spectator and outgoing quark

DY: Sivers asymmetry from interaction between spectator and incoming quark

$$f_{1T}^{\perp}(x,\boldsymbol{k}_T^2)_{\text{SIDIS}} = -f_{1T}^{\perp}(x,\boldsymbol{k}_T^2)_{\text{DY}}$$

More complicated Wilson link structures in other hard processes studied by [Bomhof, Mulders & Pijlman 2004]. (Time-reversal modified) universality between e^+e^-, DY and SIDIS shown by [Collins & Metz 2004]

At twist 3, effective T-odd distributions emerge from gluonic poles [Boer, Mulders & Teryaev 1997]. Connections between $\boldsymbol{k}_T$-dependent and twist-3 distributions deserve further study

Probing transversity: taxonomy

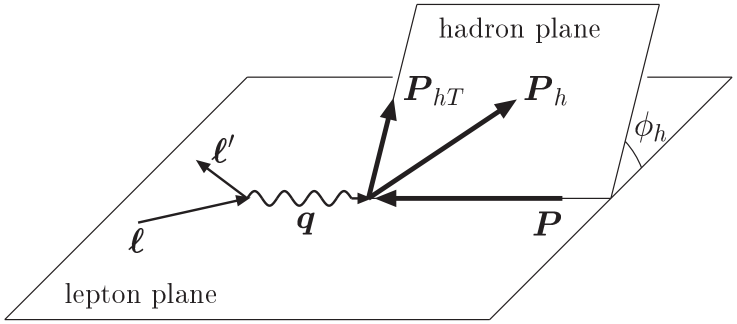
To probe transversity one needs at least one extra hadron besides the transversely polarized target

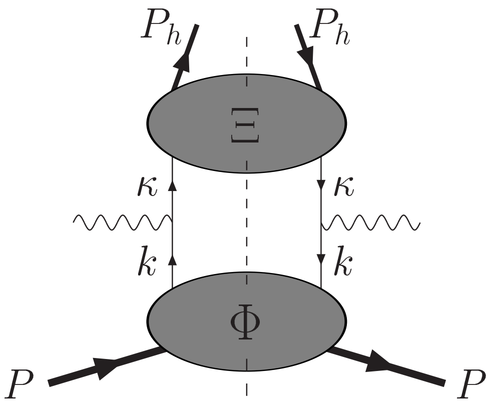
- One hadron in initial state, one in final state (ep scattering):
  - $e\,p^{\uparrow} \to e'\,\pi\,X$ single spin
  - $e\,p^{\uparrow} \to e'\,\pi^{+}\,\pi^{-}\,X$ single spin
  - $e\,p^{\uparrow} \to e'\,\Lambda^{\uparrow}\,X$ double spin
- Two hadrons in initial state (pp scattering):
  - $p^{\uparrow}\,p^{\uparrow} \to \mu^{+}\,\mu^{-}\,X$ double spin
  - $p^{\uparrow}\,p^{\uparrow} \to \gamma\,X$ double spin
  - $p\,p^{\uparrow} \to h\,X$ single spin

- Single-spin processes: easier to investigate; asymmetries arise from k_T-dependent distribution and fragmentation functions [except *]
  - $e\,p^{\uparrow} \to e'\,\pi\,X$ (Collins and Sivers effects with different angular distributions)
  - $p\,p^{\uparrow} \to h\,X$ (k_T-factorization only conjectured, not proven; mixing of various asymmetry sources)
  - $e\,p^{\uparrow} \to e'\,\pi^{+}\,\pi^{-}\,X$ [*] (collinear factorization, interference between fragmentation channels)
- Double spin processes: collinear factorization, but experimentally more difficult
  - $e\,p^{\uparrow} \to e'\,\Lambda^{\uparrow}\,X$ (fragmentation into Λ unknown)
  - $p^{\uparrow}\,p^{\uparrow} \to \mu^{+}\,\mu^{-}\,X$, $p^{\uparrow}\,p^{\uparrow} \to \gamma\,X$ (clean, but small asymmetries expected)

Semi-inclusive DIS with a transversely polarized target

$$e\,p^{\uparrow} \rightarrow e'\,h\,X$$





$$W^{\mu\nu} \sim \mathrm{Tr}\,[\Phi\,\gamma^{\mu}\,\Xi\gamma^{\nu}]$$

Ξ fragmentation matrix

Fragmentation matrix (no $\boldsymbol{\kappa}_T$): $\Xi(z) \sim D_1(z)\not{P} + \lambda_h\,G_1(z)\gamma_5\not{P} + H_1(z)\gamma_5\not{S}_T\not{P}$

In theory, the simplest access to transversity is given by the double-spin process $e\,p^{\uparrow} \to e'\,\Lambda^{\uparrow}\,X$, which involves

$$H_1(z) = \mathcal{N}_{h^{\uparrow}/q^{\uparrow}}(z) - \mathcal{N}_{h^{\uparrow}/q^{\downarrow}}(z) \qquad \text{[analog of } h_1\text{]}$$

Cross-section integrated over $\boldsymbol{P}_T$:

$$\mathrm{d}\sigma \sim (1-y)\,\cos(\phi_S + \phi_{S_\Lambda})\,h_1(x)H_1(z)$$

Polarization of Λ (measurable by studying the $\Lambda \to p\pi$ decay):

$$\mathcal{P}_\Lambda = \hat{a}(y)\,\frac{\sum_q e_q^2 h_1^q(x) H_1^q(z)}{\sum_q e_q^2 f_1^q(x) D_1^q(z)}$$

where $\hat{a}(y) = 2(1-y)/[1+(1-y)^2]$ is the QED depolarizing factor (elementary transverse asymmetry).

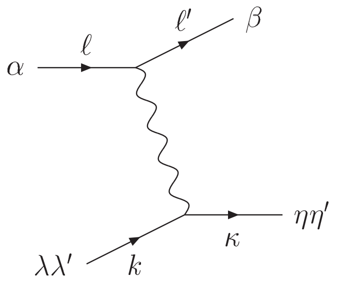
The main difficulty in predicting $\mathcal{P}_\Lambda$ is that $H_{1\Lambda}$ is completely unknown (see some models in [Anselmino, Boglione & Murgia 2000])

In single-spin processes $e\,p^{\uparrow} \rightarrow e'\,\pi^{\pm,0}\,X$ asymmetries come from transverse motion of quarks (one has to look at $\boldsymbol{P}_{hT}$ distributions)

Collinear case ($\lambda, \eta, \ldots$ helicity indices):

$$\mathrm{d}\sigma \sim \sum_{a} \sum_{\lambda\lambda'\eta\eta'} f_a(x)\,\rho_{\lambda\lambda'}\,\mathrm{d}\hat{\sigma}_{\lambda\lambda'\eta\eta'}\,\mathcal{D}^{\eta\eta'}_{h/a}(z)$$

partonic subprocess
lepton-quark scattering



Ignoring $\boldsymbol{k}_T$ and $\boldsymbol{\kappa}_T$, the dependence on the transverse spin of the target is carried by ρ_{+-}, which couples to $\mathcal{D}^{+-}_{h/a}$. Since $\mathcal{D}^{+-}_{h/a}$ vanishes for unpolarized hadrons, there is no asymmetry in pion production

Non collinear case (factorization proven by [Ji, Ma & Yuan 2004] for $P_{hT} \ll Q$)

$$\mathrm{d}\sigma \sim \sum_a \sum_{\lambda\lambda'\eta\eta'} \mathcal{P}_a(x, \boldsymbol{k}_T)\, \rho_{\lambda\lambda'} \otimes \mathrm{d}\hat{\sigma}_{\lambda\lambda'\eta\eta'} \otimes \mathcal{D}^{\eta\eta'}_{h/a}(z, \boldsymbol{\kappa}_T)$$

Single-spin transverse asymmetry are due to

- spin asymmetry of transversely polarized quarks fragmenting into an unpolarized hadron (Collins effect) [non vanishing $\mathcal{D}^{+-}_{h/a}$]
- azimuthal asymmetry of unpolarized quarks inside the transversely polarized proton (Sivers effect) [target transverse spin dependence in ρ_{++}, ρ_{--}]

Fragmentation of transversely polarized quarks into an unpolarized hadron:

$$\mathcal{N}_{h/q^\uparrow}(z, \boldsymbol{P}_{hT}) = D_1(z, \boldsymbol{P}_{hT}^2) + \frac{(\hat{\boldsymbol{\kappa}}_T \times \boldsymbol{P}_{hT}) \cdot \boldsymbol{S}_{qT}}{z M_h} H_1^\perp(z, \boldsymbol{P}_{hT}^2)$$

$$\mathcal{N}_{h/q^\uparrow}(z, \boldsymbol{P}_{hT}) - \mathcal{N}_{h/q^\downarrow}(z, \boldsymbol{P}_{hT}) = \frac{(\hat{\boldsymbol{\kappa}}_T \times \boldsymbol{P}_{hT}) \cdot \boldsymbol{S}_{qT}}{z M_h} H_1^\perp(z, \boldsymbol{P}_{hT}^2)$$

$H_1^\perp(z, \boldsymbol{P}_{hT}^2)$ is the Collins fragmentation function, a T-odd function (not forbidden by time reversal invariance due to final-state interactions)

Distribution of unpolarized quarks inside a transversely polarized proton:

$$\mathcal{P}_{q/p^\uparrow}(x, \boldsymbol{k}_T) - \mathcal{P}_{q/p^\uparrow}(x, -\boldsymbol{k}_T) = \frac{(\boldsymbol{k}_T \times \hat{\boldsymbol{P}}) \cdot \boldsymbol{S}_T}{M} f_{1T}^\perp(x, \boldsymbol{k}_T^2)$$

$f_{1T}^\perp(x, \boldsymbol{k}_T^2)$ is the Sivers distribution function

Transversely polarized pion leptoproduction cross section:

$$
\begin{aligned}
\mathrm{d}\sigma \quad \sim \quad & A(y)\,\mathcal{I}\left[\frac{\boldsymbol{\kappa}_T\cdot\hat{\boldsymbol{P}}_{hT}}{M_h}\,h_1^a\,H_1^{\perp a}\right]\sin(\phi_h+\phi_S)\\
& + B(y)\,\mathcal{I}\left[\frac{\boldsymbol{k}_T\cdot\hat{\boldsymbol{P}}_{hT}}{M_h}\,f_{1T}^{\perp a}\,D_1^a\right]\sin(\phi_h-\phi_S)\\
& + C(y)\,\mathcal{I}\left[\frac{4(\boldsymbol{k}_T\cdot\hat{\boldsymbol{P}}_{hT})^2(\hat{\boldsymbol{P}}_{hT}\cdot\boldsymbol{\kappa}_T)-2(\boldsymbol{k}_T\cdot\hat{\boldsymbol{P}}_{hT})(\boldsymbol{k}_T\cdot\boldsymbol{\kappa}_T)-\boldsymbol{k}_T^2(\boldsymbol{\kappa}_T\cdot\hat{\boldsymbol{P}}_{hT})}{2M^2M_h}\,h_{1T}^{\perp a}\,H_1^{\perp a}\right]\\
& \quad \times\sin(3\phi_h-\phi_S)
\end{aligned}
$$

$$
\mathcal{I}[f\;D]\equiv\int\mathrm{d}^2\boldsymbol{k}_T\int\mathrm{d}^2\boldsymbol{\kappa}_T\,\delta^2(\boldsymbol{k}_T-\boldsymbol{P}_{hT}/z-\boldsymbol{\kappa}_T)\,f(x,\boldsymbol{k}_T^2)\,D(z,\boldsymbol{\kappa}_T^2)
$$

Assuming Gaussian dependence on transverse momenta, $\mathcal{I}[f\;D]\propto f\;D$

Variety of angular distributions (two independent physical angles):

- Collins effect: $\sin(\phi_h + \phi_S)$ and [if $\boldsymbol{k}_T \neq 0$] $\sin(3\phi_h - \phi_S)$
- Sivers effect: $\sin(\phi_h - \phi_S)$

Disentangled by taking azimuthal moments, for instance:

$$\langle \sin(\phi_h + \phi_S) \rangle \equiv \frac{\int \mathrm{d}\phi_h \, \mathrm{d}\phi_S \; \sin(\phi_h + \phi_S) \, [\mathrm{d}\sigma(\phi_h, \phi_S) - \mathrm{d}\sigma(\phi_h, \phi_S + \pi)]}{\int \mathrm{d}\phi_h \, \mathrm{d}\phi_S \, [\mathrm{d}\sigma(\phi_h, \phi_S) + \mathrm{d}\sigma(\phi_h, \phi_S + \pi)]}$$

Other occurrences of transversity (and related functions) in $ep^{\uparrow} \to e'\pi X$

Integrated cross-sections at twist 3:

- Unpolarized lepton beam:

  $\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} \sim h_1(x)\,\widetilde{H}(z)$

  h_1 couples to the twist-3 fragmentation function $\widetilde{H}$

- Longitudinally polarized lepton beam:

  $\mathrm{d}\sigma^{\rightarrow\uparrow} - \mathrm{d}\sigma^{\rightarrow\downarrow} \sim h_1(x)\,\widetilde{E}(z)$

  h_1 couples to the twist-3 fragmentation function $\widetilde{E}$

$\boldsymbol{P}_{hT}$ distributions at twist 3:

- Unpolarized lepton beam:

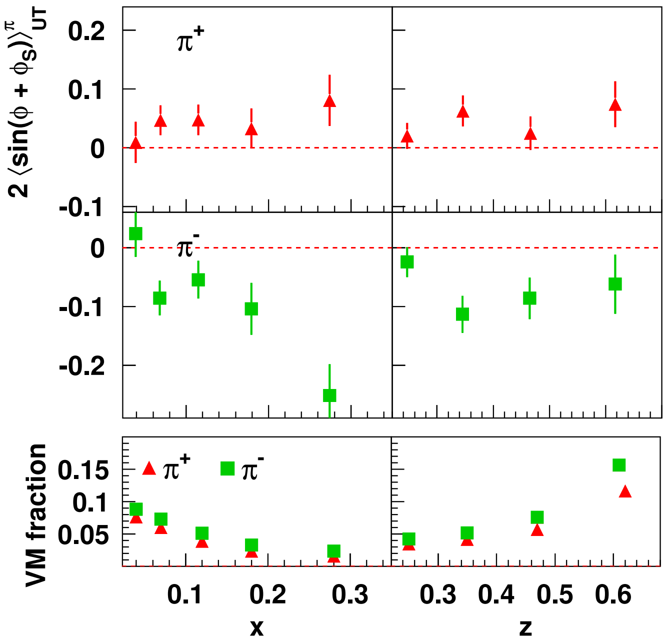
  $\sin(2\phi_h - \phi_S)$ asymmetry from coupling of various twist-3 distributions to Collins function $H_1^{\perp}$

HERMES results on single-spin asymmetries in SIDIS

[Airapetian et al. 2004]

$$\langle x \rangle = 0.09\,, \quad \langle z \rangle = 0.36\,, \quad \langle Q^2 \rangle = 2.4\ \text{GeV}^2\,, \quad \langle P_{hT} \rangle = 0.4\ \text{GeV}$$

Collins asymmetries



$A_T^{\pi^+} > 0$, $A_T^{\pi^-} < 0$: consistent with h_1^u positive, h_1^d negative. But $|A_T^{\pi^-}| \gtrsim |A_T^{\pi^+}|$, whereas one expects from models $|h_1^d| \ll |h_1^u|$

Possible explanation: large unfavored Collins functions

$$\pi^+ : \quad 4\, h_1^u\, H_1^{\perp \mathsf{fav}} + h_1^d\, H_1^{\perp \mathsf{unf}}$$
$$\pi^- : \quad h_1^d\, H_1^{\perp \mathsf{fav}} + 4\, h_1^u\, H_1^{\perp \mathsf{unf}}$$

π^- data seem to require $H_1^{\perp \mathsf{unf}} \approx -H_1^{\perp \mathsf{fav}}$

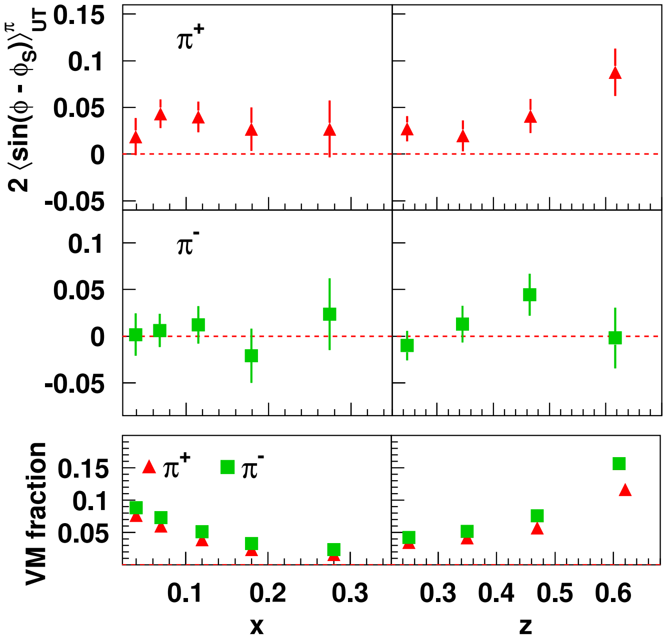
Independent information on $H_1^{\perp}$ is very important:

$\Rightarrow e^+e^-$ data from b-factories [Belle]

Other HERMES results on Collins asymmetry:

- Preliminary data on π^0 asymmetry are quite controversial (largely negative $A_T^{\pi^0}$, in conflict with expectations based on isospin invariance)
- $\sin(3\phi_h - \phi_S)$ contribution (due to $h_{1T}^{\perp}$ coupling to Collins function) negligible within errors

Sivers asymmetries



$A_T^{\pi^+} > 0$, $A_T^{\pi^-} \sim 0$: evidence for non vanishing $f_{1T}^{\perp}$, but more precise data are needed

COMPASS preliminary results on Collins asymmetry with a deuteron target presented at DIS 04

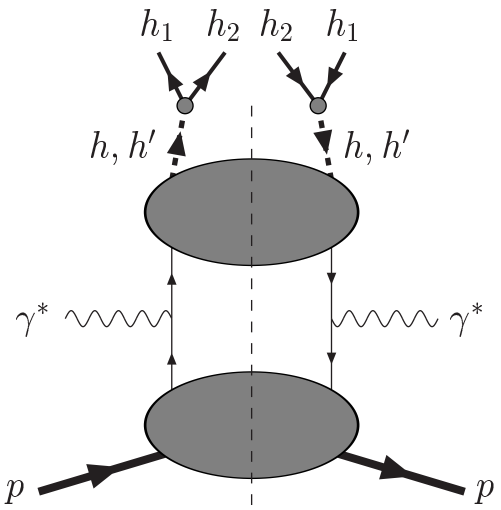
Kinematics:

$$x \lesssim 0.1 \quad \langle Q^2 \rangle = 2.4\,\mathsf{GeV}^2\,, \quad \langle P_{hT}^2 \rangle = 0.3\,\mathsf{GeV}^2\,,$$

The single-spin asymmetry $A_T^{\pi^\pm}$ is found to be compatible with zero (as expected)

Transversity in two-particle leptoproduction

[Collins & Ladinski 1994, Jaffe, Jin & Tang 1998, Bianconi et al. 2000, Bacchetta & Radici 2003]



$$e + p^{\uparrow} \to e' + h_1 + h_2 + X$$

$$M_h^2 = (P_1 + P_2)^2, \quad R = \tfrac{1}{2}(P_1 - P_2)$$

$$z = z_1 + z_2, \quad \xi = z_1/z = 1 - z_2/z$$

Probes angular correlations of the form $(\boldsymbol{P}_1 \times \boldsymbol{P}_2) \cdot \boldsymbol{S}_T = (\boldsymbol{P}_{hT} \times \boldsymbol{R}_T) \cdot \boldsymbol{S}_T$

Transverse polarization of fragmenting quark gives rise to orbital relative motion of h_1 and h_2

Cross section (collinear factorization):

$$d\sigma \sim \sum_q e_q^2 h_1^q(x)\, I_q(z, M_h^2)\, \sin(\phi_S + \phi_R)$$

$I_q(z, M_h^2)$ is an interference fragmentation function, arising from interference between different production channels

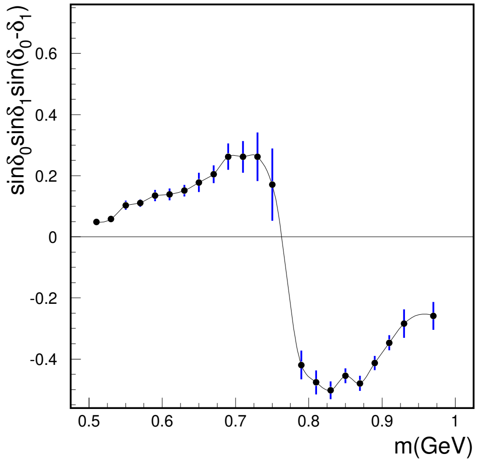
Consider $\pi^+\pi^-$ production via σ, ρ resonance formation [Jaffe, Jin & Tang 1998]:

$$|\pi^+\pi^-\rangle = e^{i\delta_0(M_h)}\,|\sigma\rangle + e^{i\delta_1(M_h)}\,|\rho\rangle$$

The interference fragmentation function has the form

$$I(z, M_h^2) \sim \sin\delta_0\ \sin\delta_1\ \sin(\delta_0 - \delta_1)\ \hat{I}(z)$$

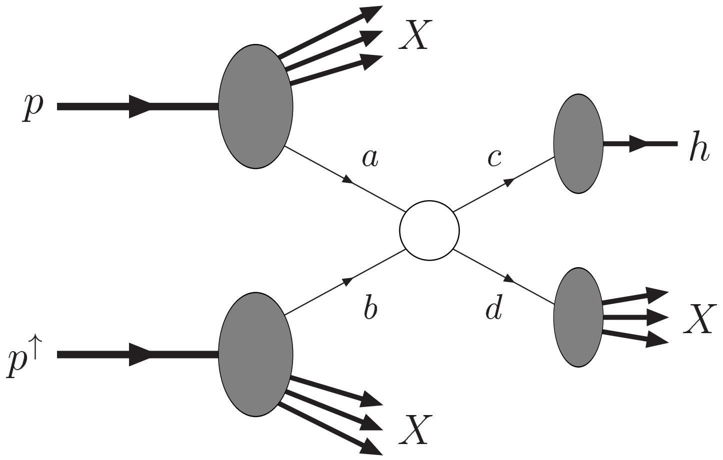
and changes sign around $M_h = m_\rho$.



In the model of [Radici, Jakob & Bianconi 2002] $I(z, M_h^2)$ has a Breit-Wigner shape.

⇒ Preliminary HERMES data

Single-spin asymmetries in hadroproduction



$$p + p^\uparrow \to h + X$$

$$x_F = \frac{2\,P_L}{\sqrt{s}}$$

Single transverse asymmetry : $A_T = \dfrac{\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow}{\mathrm{d}\sigma^\uparrow + \mathrm{d}\sigma^\downarrow}$

In leading-twist QCD one has $A_T = 0$ (Kane, Pumplin & Repko 1978)

A non-zero asymmetry is generated either by intrinsic k_T effects, or by higher twist effects

Collinear factorization formula ($\alpha, \gamma \ldots$ helicity indices):

$$\mathrm{d}\sigma \sim \sum_{abc} \sum_{\alpha\alpha'\gamma\gamma'} \rho^{a}_{\alpha'\alpha}\, f_a(x_a) \otimes f_b(x_b) \otimes \mathrm{d}\hat{\sigma}_{\alpha\alpha'\gamma\gamma'} \otimes \mathcal{D}^{\gamma'\gamma}_{h/c}(z)$$

For an unpolarized hadron h one has $\mathcal{D}^{\gamma'\gamma}_{h/c} \propto \delta_{\gamma\gamma'}$ and therefore, by helicity conservation, $\alpha = \alpha'$: no dependence on the spin of the target → $A_T = 0$

Introduce transverse motion of quarks: non collinear factorization (conjectured):

$$\begin{aligned} \mathrm{d}\sigma \quad \sim \quad & \sum_{abc} \sum_{\alpha\alpha'\beta\beta'\gamma\gamma'} \rho^{a}_{\alpha'\alpha}\, \mathcal{P}_a(x_a, \boldsymbol{k}_T) \otimes \rho^{b}_{\beta'\beta}\, \mathcal{P}_b(x_b, \boldsymbol{k}'_T) \\ & \otimes \mathrm{d}\hat{\sigma}_{\alpha\alpha'\beta\beta'\gamma\gamma'} \otimes \mathcal{D}^{\gamma'\gamma}_{h/c}(z, \boldsymbol{\kappa}_T) \end{aligned}$$

Three sources of SSA: Collins effect, Sivers effect, Boer-Mulders effect

Collins effect:

$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} \quad \sim \quad \sum_{abc} \left[h_1(x_a, \boldsymbol{k}_T^2) + \frac{\boldsymbol{k}_T^2}{M^2}\, h_{1T}^{\perp}(x_a, \boldsymbol{k}_T^2)\right] \otimes f_1(x_b, \boldsymbol{k}_T'^{\,2}) \otimes \Delta_{TT}\hat{\sigma}(a^{\uparrow} b \to c^{\uparrow} d) \otimes H_1^{\perp}(z, \boldsymbol{\kappa}_T^2)\,,$$

In many calculations, only $\boldsymbol{\kappa}_T$ is considered (no $h_{1T}^{\perp}$ term). A complete treatment of transverse momenta in [Anselmino et al. 2004]

Sivers effect:

$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} \sim f_{1T}^{\perp}(x_a, \boldsymbol{k}_T^2) \otimes f_1(x_b, \boldsymbol{k}_T'^{\,2}) \otimes \mathrm{d}\hat{\sigma}(ab \to cd) \otimes D_1(z, \boldsymbol{\kappa}_T^2)\,,$$

Boer-Mulders effect:

$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} \quad \sim \quad \sum_{abc} \left[h_1(x_a, \boldsymbol{k}_T^2) + \frac{\boldsymbol{k}_T^2}{M^2}\, h_{1T}^{\perp}(x_a, \boldsymbol{k}_T^2)\right] \otimes h_1^{\perp}(x_b, \boldsymbol{k}_T'^{\,2}) \otimes \Delta_{TT}\hat{\sigma}(a^{\uparrow} b^{\uparrow} \to cd) \otimes D_1(z, \boldsymbol{\kappa}_T^2)\,,$$

The three sources of SSA are entangled in $pp^{\uparrow} \to \pi X$ (the only physical angle is $\phi_h - \phi_S$). To disentangle them one would need less inclusive measurements (for instance, hadron + jet) [Teryaev].

SSA from higher twists:

$$\begin{aligned} \mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} \quad \sim \quad & G_F(x_a, y_a) \otimes f_1(x_b) \otimes \mathrm{d}\hat{\sigma} \otimes D_1(z) \\ & + h_1(x_a) \otimes E_F(x_b, y_b) \otimes \mathrm{d}\hat{\sigma}' \otimes D_1(z) \\ & + h_1(x_a) \otimes f_1(x_b) \otimes \mathrm{d}\hat{\sigma}'' \otimes D^{(3)}(z) \end{aligned}$$

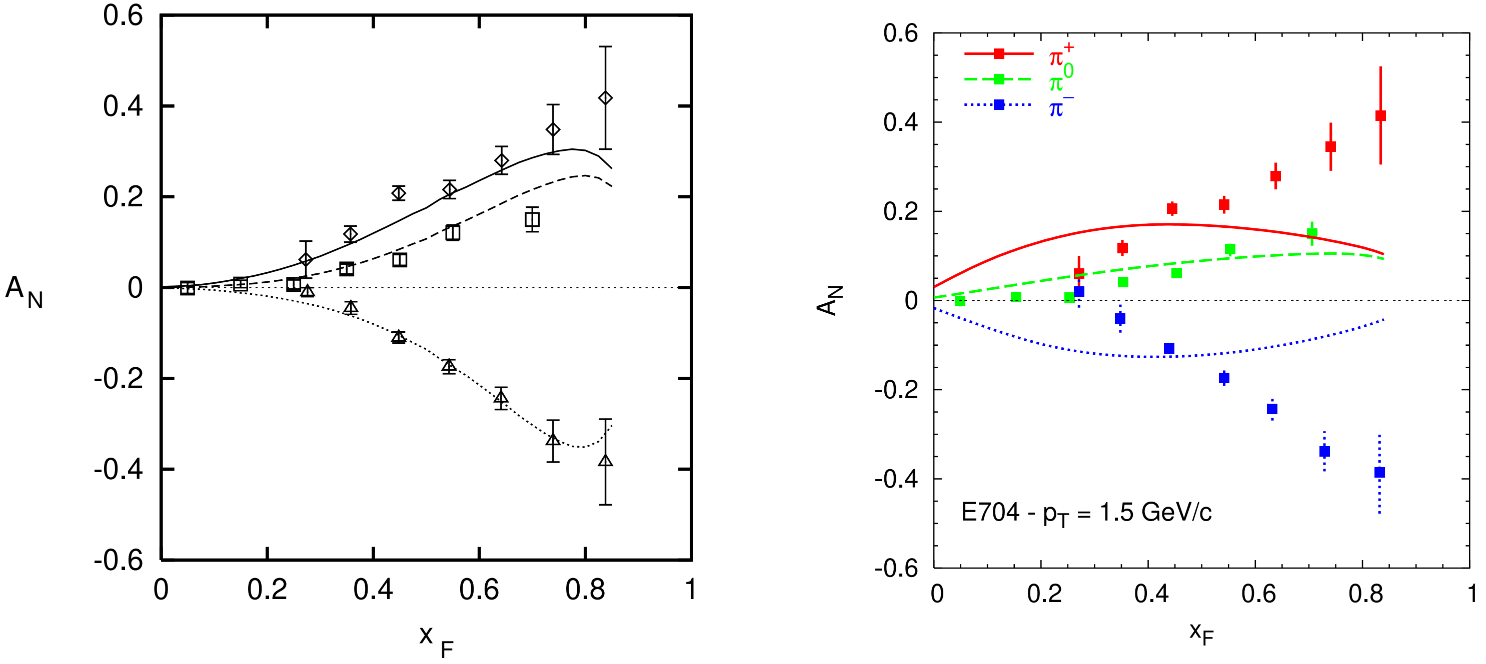
$G_F(x,y)$ and $E_F(x,y)$ are quark-gluon correlation functions

$D^{(3)}(z)$ is a twist-3 fragmentation function

E704 results on single-spin asymmetries in $pp^{\uparrow} \to \pi X$

[D.L. Adams et al. 1991, Bravar et al. 1996]

$$\sqrt{s} = 19.4\,\text{GeV}\,, \qquad 0.2\,\text{GeV} < \langle P_{\pi T} \rangle < 2.0\,\text{GeV}$$



Left: Collins effect with no quark transverse momentum inside the target [Anselmino, Boglione & Murgia 1999]

Right: Collins effect with all transverse momenta taken into account [Anselmino et al. 2004].

The suppression of Collins effect in $pp^{\uparrow}$ is due to kinematic phases arising from cross sections of non collinear partonic scattering

$$\sin(\alpha)\; h_1 H_1^{\perp} + \sin(\beta)\; h_{1T}^{\perp} H_1^{\perp}$$

Nothing to do with the magnitude of Collins function

What about Collins effect in $ep^{\uparrow}$?

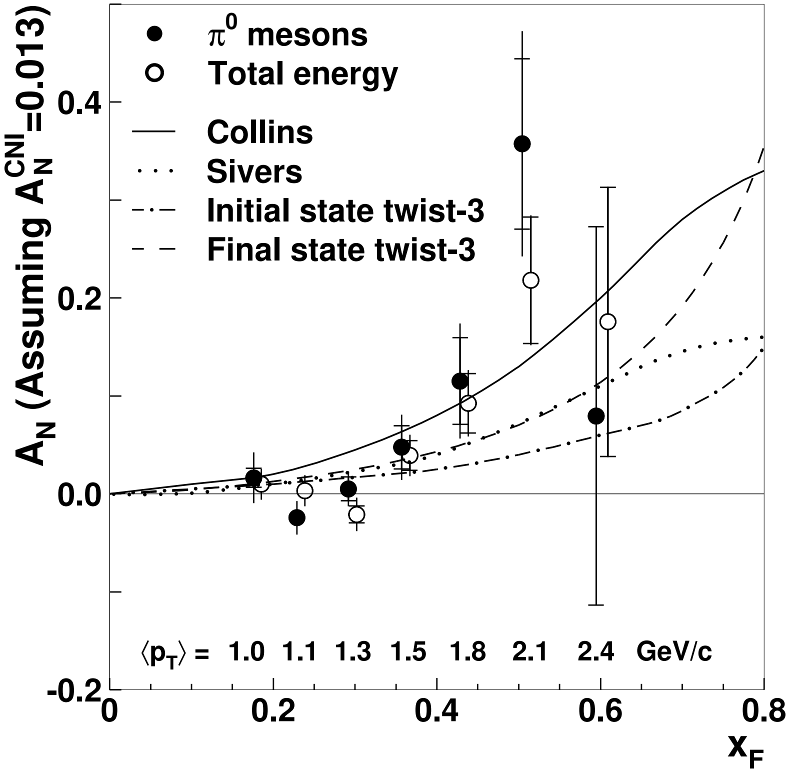
$$\sin(\phi_h - \phi_S)\; h_1 H_1^{\perp} + \sin(3\phi_h - \phi_S)\; h_{1T}^{\perp} H_1^{\perp}$$

In this case there are two physical angles and therefore two different angular distributions

STAR results on single-spin asymmetries in $pp^{\uparrow} \to \pi^0 X$

[J. Adams et al. 2004]

$$\sqrt{s} = 200\,\text{GeV}\,, \qquad 1.0\,\text{GeV} < \langle P_{\pi T} \rangle < 2.4\,\text{GeV}$$



Well described by: Collins effect, Sivers effect [Anselmino et al.] and higher-twist distribution or fragmentation functions [Qiu & Sterman, Kanazawa & Koike]

E704 vs. (STAR + PHENIX)

E704 : $\sqrt{s} = 19.4\,\text{GeV}\,, \quad 9^o < \theta < 67^o$

STAR : $\sqrt{s} = 200\,\text{GeV}\,, \quad \theta = 2.6^o$

PHENIX : $\sqrt{s} = 200\,\text{GeV}\,,$ large $P_{\pi T}$, $\theta \sim 90^o$ (central region) $A_T \approx 0$

[For fixed P_T and $\sqrt{s}$, large (small) $x_F \Leftrightarrow$ small (large) θ]

- Unpolarized cross sections: collinear pQCD correctly reproduce STAR and PHENIX data, but fails to reproduce E704 data (especially at small θ).

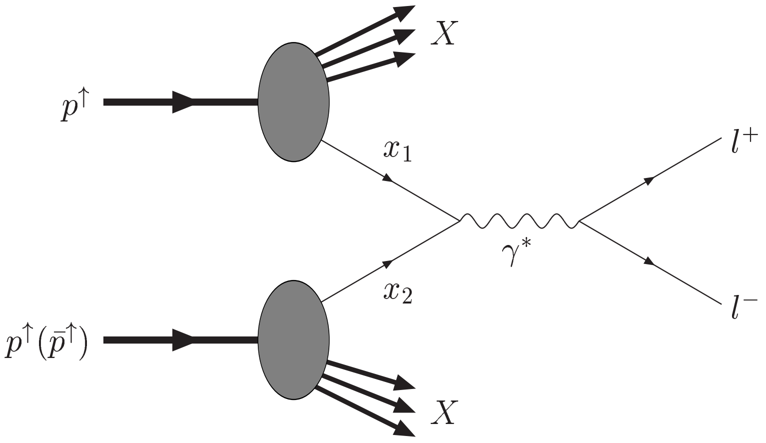
  Conclusion by [Bourrely & Soffer 2004]: “STAR and E704 asymmetries are different phenomena; E704 asymmetry cannot be attributed to pQCD”

  However, higher twist effects might be important ($P_{\pi T}$ is not so large)

- [D'Alesio & Murgia 2004]: quark transverse momenta considerably improve agreement with unpolarized cross sections at small $\sqrt{s}$

Transversity in Drell-Yan processes

$$p^{\uparrow} + p^{\uparrow} \to \mu^{+} + \mu^{-} + X$$



$x_1 x_2 = \frac{M^2}{s} \equiv \tau$

$y = \frac{1}{2} \ln \frac{x_1}{x_2}$ (rapidity of γ^*)

M^2 invariant mass of $l^+ l^-$

Double transverse asymmetry : $A_{TT}^{DY} = \dfrac{d\sigma^{\uparrow\uparrow} - d\sigma^{\uparrow\downarrow}}{d\sigma^{\uparrow\uparrow} + d\sigma^{\uparrow\downarrow}}$

At leading order ($q\bar{q}$ annihilation):

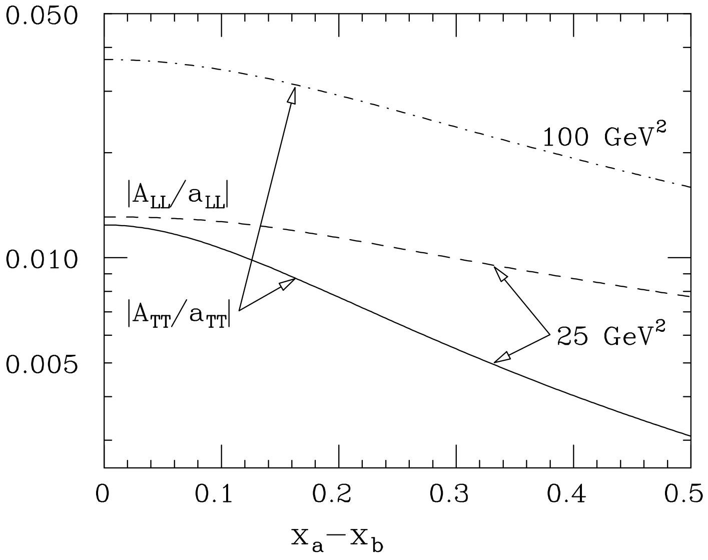
$$A_{TT}^{DY} \sim \frac{\sin^2\theta \, \cos 2\phi}{1 + \sin^2\theta} \, \frac{\sum_q e_q^2 h_1^q(x_1, M^2) \bar{h}_1^q(x_2, M^2) + [1 \leftrightarrow 2]}{\sum_q e_q^2 f_1^q(x_1, M^2) \bar{f}_1^q(x_2, M^2) + [1 \leftrightarrow 2]}$$

Predictions for $A_{TT}^{DY}(pp)$

LO at $\sqrt{s} = 100$ GeV

$h_1 = g_1$ at $Q_0^2 = 0.23$ GeV^2

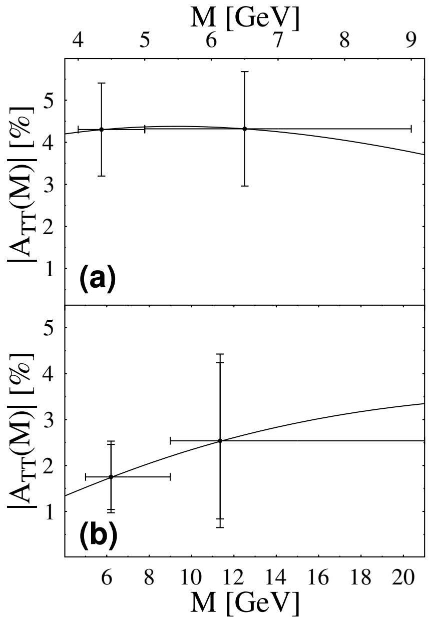
[VB, Calarco & Drago 1997]



NLO at $\sqrt{s} = 40$ GeV (a), 200 GeV (b)

Soffer bound saturated at Q_0

[Martin et al. 1998]



At RHIC energies, very small asymmetries: $A_{TT}^{DY}(pp) \sim 1 - 2\%$

Similar results for prompt photon production: $A_{TT}^{\gamma}(pp) \sim 1\,\%$ [Mukherjee, Stratmann & Vogelsang 2003]

$\bar{p}^{\uparrow} p^{\uparrow} \to \mu^+ \mu^- (J/\psi) X$ at GSI

$A_{TT}^{DY}(pp)$ at RHIC energies is small because:

1. antiquark transversity distributions are small
2. $\sqrt{s} = 200$ GeV, $M < 10$ GeV $\Rightarrow x_1 x_2 = \frac{M^2}{s} < 2.5 \times 10^{-3}$, and at small x transversity evolution is suppressed

[PAX proposal]: polarized antiprotons colliding on polarized protons at HESR of GSI

$$30\ \text{GeV}^2 \lesssim s \lesssim 45\,\text{GeV}^2\,, \quad M \gtrsim 2\,\text{GeV}\,, \quad x_1 x_2 \gtrsim 0.1$$

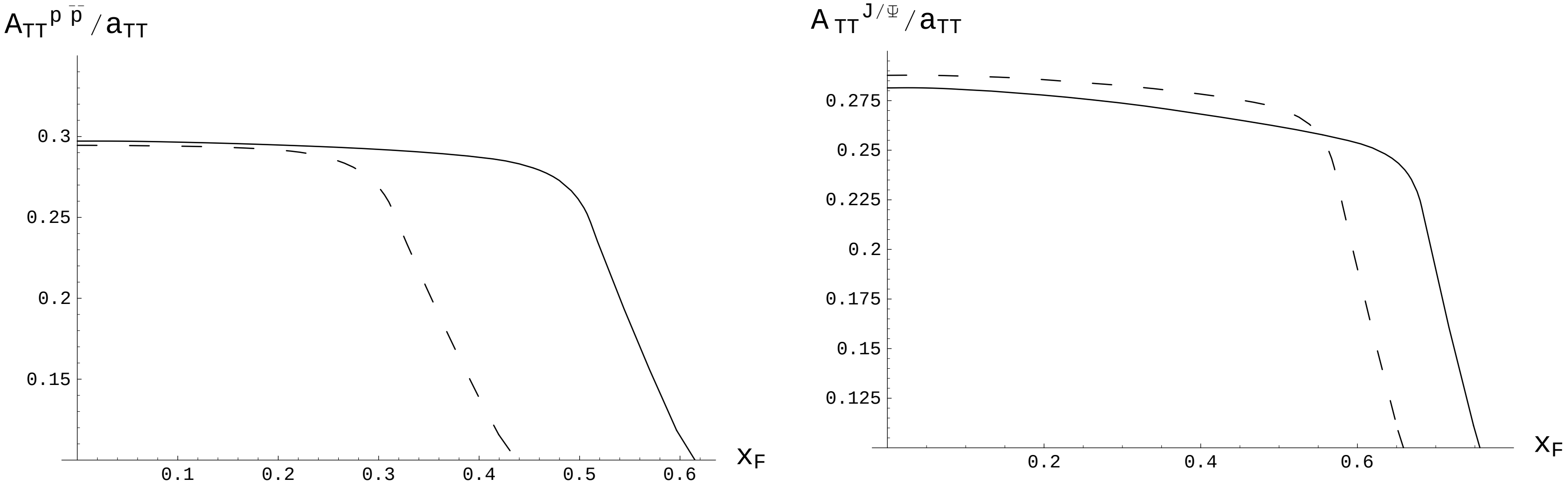
$$A_{TT}^{DY}(\bar{p}p) \sim \frac{\sum_a e_a^2\,[h_1^a(x_1, M^2) h_1^a(x_2, M^2) + \bar{h}_1^a(x_1, M^2)\bar{h}_1^a(x_2, M^2)]}{\sum_a e_a^2\,[f_1^a(x_1, M^2) f_1^a(x_2, M^2) + \bar{f}_1^a(x_1, M^2)\bar{f}_1^a(x_2, M^2)]}$$

Large asymmetry (valence-valence combinations), but small rate

$\Rightarrow J/\psi$ production [Anselmino, VB, Drago, Nikolaev 2004]

- Comparison of J/ψ production in $\bar{p}p$ and pp collisions at $s = 80$ GeV^2 shows dominance of $\bar{q}q$ annihilation
- Assume that J/ψ carries over the polarization of the $\bar{c}c$ pair

$$A_{TT}^{J/\psi} \sim \frac{h_1^u(x_1, M_\psi^2) h_1^u(x_2, M_\psi^2)}{f_1^u(x_1, M_\psi^2) f_1^u(x_2, M_\psi^2)}$$



DY at $M = 4$ GeV

J/ψ production ($M = 3$ GeV)

(solid: $s = 45$ GeV^2, dashed: $s = 30$ GeV^2)

$A_{TT}^{J/\psi} \sim 0.3$ (similar results by [Efremov, Goeke & Schweitzer 2004])

Summary and conclusions

- Transversity is one of the hottest topics in high-energy spin physics
- From the theoretical point of view, transversity is well understood:
  - QCD evolution known at NLO
  - h_1 and tensor charges sizeable (models, lattice, ...)
  - Related k_T-dependent distributions better known
- Phenomenology of transversity:
  - Single-spin processes are easier to explore but involve unknown functions and require a clear separation of different