

**Can the Collins mechanism explain
the large transverse single spin asymmetries
observed in $p^\uparrow p \rightarrow \pi X$?**

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[In coll. with: *M. Anselmino, M. Boglione, E. Leader, S. Melis, F. Murgia*]
(hep-ph/0408356 + work in progress)

Outline

- Generalized pQCD approach with **spin and k_{\perp} -effects** in distribution and fragmentation functions and **elementary dynamics**;
- SSA in $pp \Rightarrow \pi X$: **theory - helicity formalism**;
- SSA in $pp \Rightarrow \pi X$: **phenomenology of the Collins effect**;
- Conclusions and outlook

Unpolarized cross sections: k_{\perp} -factorization *ansatz*

- The collinear pQCD factorization formula for $AB \rightarrow CX$ reads

$$d\sigma^{AB \rightarrow CX} = \sum_{a,b,c,d} f_{a/A}(x_a) \otimes f_{b/B}(x_b) \otimes d\hat{\sigma}^{ab \rightarrow cd}(\hat{s}, \hat{t}) \otimes D_{C/c}(z)$$

$$\hat{s} = sx_a x_b, \quad \hat{t} = tx_a/z, \quad \hat{u} = ux_b/z$$

- Including transverse momenta we have:

$$\Rightarrow \sum_{a,b,c,d} \hat{f}_{a/A}(x_a, \mathbf{k}_{\perp a}) \otimes \hat{f}_{b/B}(x_b, \mathbf{k}_{\perp b}) \otimes d\hat{\sigma}^{ab \rightarrow cd}(\hat{s}, \hat{t}) \otimes \hat{D}_{C/c}(z, \mathbf{k}_{\perp C})$$

$$\hat{s} \rightarrow \hat{s} \phi_s(\mathbf{k}_{\perp a}, \mathbf{k}_{\perp b}), \quad \hat{t} \rightarrow \hat{t} \phi_t(\mathbf{k}_{\perp a}, \mathbf{k}_{\perp C}), \quad \hat{u} \rightarrow \hat{u} \phi_u(\mathbf{k}_{\perp b}, \mathbf{k}_{\perp C})$$

+ Jacobian in FF + Cahn effect + Gaussian k_{\perp} -depend.; + factoriz. ...

see *U. D., F. Murgia*: hep-ph/0408092, PRD in press & F. Murgia talk

k_{\perp} - effects

1. k_{\perp} alters $\langle x \rangle$: small shift but important at large x where PDF are varying rapidly.
 → *enhanced higher-twist* effect (up to a factor of 10 in a cross section);
2. *the partonic scattering angle* in the pp c.m. frame *might be much smaller* than the hadronic production angle, thus enhancing the moderately large p_T inclusive production of particles;
3. *spin and k_{\perp} -effects generated by soft mechanisms: SSA ($pp \rightarrow hX$)*
 - (a) *Sivers PDF:* $\mathbf{S} \cdot (\mathbf{P} \times \mathbf{k}_{\perp q}) = \sin(\phi_S - \phi_k)$
 - (b) *Collins FF:* $\mathbf{s}_q \cdot (\mathbf{p}_q \times \mathbf{k}_{\perp h})$
 - (c) *Boer-Mulders PDF:* $\mathbf{s}_q \cdot (\mathbf{P} \times \mathbf{k}_{\perp q})$
 - (d) *polarizing FF:* $\mathbf{S}_h \cdot (\mathbf{p}_q \times \mathbf{k}_{\perp h})$

Polarized cross sections: Helicity formalism

◇ **Helicity density matrices**, in the \mathbf{k}_\perp -factorization scheme, to describe parton spin states for a polarized cross section:

$$d\sigma^{A,S_A+B,S_B \rightarrow C+X} = \sum_{a,b,c,d,\{\lambda\}} \rho_{\lambda_a,\lambda'_a}^{a/A,S_A} \hat{f}_{a/A,S_A}(x_a, \mathbf{k}_\perp a) \otimes \rho_{\lambda_b,\lambda'_b}^{b/B,S_B} \hat{f}_{b/B,S_B}(x_b, \mathbf{k}_\perp b) \\ \otimes \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} \hat{M}_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}^* \otimes \hat{D}_{\lambda_c,\lambda'_c}^{\lambda_C,\lambda'_C}(z, \mathbf{k}_\perp C)$$

- $\rho_{\lambda_a,\lambda'_a}^{a/A,S_A}$: helicity density matrix of parton a inside hadron A with polarization S_A
- $\frac{d\hat{\sigma}^{ab \rightarrow cd}}{d\hat{t}} \simeq \sum_{\lambda_a,\lambda_b,\lambda_c,\lambda_d} |\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}|^2$ (scattering amplitudes)
- $\hat{D}_{\lambda_c,\lambda'_c}^{\lambda_C,\lambda'_C} \equiv \rlap{-}\int_{X,\lambda_X} \hat{\mathcal{D}}_{\lambda_C,\lambda_X;\lambda_c} \hat{\mathcal{D}}_{\lambda'_C,\lambda_X;\lambda'_c}^*$
i.e. the product of *fragmentation amplitudes* for the $c \rightarrow C + X$ process

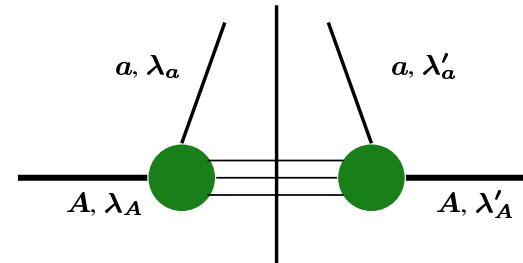
Helicity density matrices: phases from soft parts

◇ PDF at L.O. (and twist two) as the inclusive cross section for $A \rightarrow a + X$.

Helicity density matrix of parton $a \leftrightarrow$ to the helicity density matrix of hadron A .

$$\rho_{\lambda_a, \lambda'_a}^{a/A, S_A} \hat{f}_{a/A, S_A}(x_a, \mathbf{k}_{\perp a}) = \sum_{\lambda_A, \lambda'_A} \rho_{\lambda_A, \lambda'_A}^{A, S_A} \hat{F}_{\lambda_A, \lambda'_A}^{\lambda_a, \lambda'_a}$$

$$\hat{F}_{\lambda_A, \lambda'_A}^{\lambda_a, \lambda'_a} \equiv \int_{X_A, \lambda_X} \hat{\mathcal{F}}_{\lambda_a, \lambda_X; \lambda_A} \hat{\mathcal{F}}_{\lambda'_a, \lambda_X; \lambda'_A}^*$$



$$\hat{\mathcal{F}}_{\lambda_a, \lambda_X; \lambda_A}(x_a, \mathbf{k}_{\perp a}) = \mathcal{F}_{\lambda_a, \lambda_X; \lambda_A}(x_a, k_{\perp a}) \exp[i\lambda_A \phi_a]$$

$$\Rightarrow \hat{F}_{\lambda_A, \lambda'_A}^{\lambda_a, \lambda'_a}(x_a, \mathbf{k}_{\perp a}) = F_{\lambda_A, \lambda'_A}^{\lambda_a, \lambda'_a}(x_a, k_{\perp a}) \exp[i(\lambda_A - \lambda'_A) \phi_a]$$

◇ and same for FF (pion case): $\hat{D}_{\lambda_c, \lambda'_c}^{\pi} = D_{\lambda_c, \lambda'_c}^{\pi} \exp[i(\lambda_c - \lambda'_c) \phi_{\pi}^H]$

Partonic interpretation: a few examples

Spin and k_{\perp} -dependent distribution and fragmentation functions of leading twist:

[vs. notation of P.J. Mulders, R.D. Tangerman *Nucl. Phys.* **B461** (1996)]

- $\hat{f}_{q/p} = \hat{F}_{+,+}^{+,+} + \hat{F}_{-,-}^{-,-}$: unpolarized PDF
- $\Delta \hat{f}_{q/p} = \hat{F}_{+,+}^{+,+} - \hat{F}_{-,-}^{-,-}$: long. polariz. PDF
- $\hat{D}_{\pi/q} = \hat{D}_{++}^{\pi}$: unpolarized FF
- $\hat{f}_{q/p\uparrow} - \hat{f}_{q/p\downarrow} \equiv \Delta^N f_{q/p\uparrow} = 4\text{Im}\hat{F}_{+,-}^{+,+} \propto (k_{\perp}/M) f_{1T}^{\perp}$: Sivers funct.
- $\hat{f}_{q\uparrow/p} - \hat{f}_{q\downarrow/p} \equiv \Delta^N f_{q\uparrow/p} = 2\text{Im}\hat{F}_{+,+}^{+,-} \propto (k_{\perp}/M) h_1^{\perp}$: Boer-Mulders funct.
- $\hat{D}_{h/q\uparrow} - \hat{D}_{h/q\downarrow} \equiv \Delta^N D_{h/q\uparrow} = 2\text{Im}D_{+,-}^h \propto (k_{\perp}/M) H_1^{\perp}$: Collins funct.

and gluons...

see also hep-ph/0410050, (“the Trento conventions”)

A. Bacchetta, U.D., M. Diehl, A. Miller

The master formula

$$d\sigma^{A,S_A+B,S_B \rightarrow C+X} = \sum_{a,b,c,d,\{\lambda\}} \rho_{\lambda_A,\lambda'_A}^{A,S_A} \hat{F}_{\lambda_A,\lambda'_A}^{\lambda_a,\lambda'_a}(\mathbf{k}_{\perp a}) \otimes \rho_{\lambda_B,\lambda'_B}^{B,S_B} \hat{F}_{\lambda_B,\lambda'_B}^{\lambda_b,\lambda'_b}(\mathbf{k}_{\perp b}) \\ \otimes \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} \hat{M}_{\lambda'_c,\lambda'_d;\lambda'_a,\lambda'_b}^* \otimes \hat{D}_{\lambda_c,\lambda'_c}^{\lambda_C,\lambda'_C}(\mathbf{k}_{\perp C})$$

On summing over $\{\lambda's\}$ we get all combinations of spin and \mathbf{k}_{\perp} - mechanisms in Single and Double Spin Asymmetries for unpolarized particle production.

Notice: even for $S_A = S_B = 0 \rightarrow \rho_{\lambda_A,\lambda'_A}^{A,S_A} = (1/2)\delta_{\lambda_A,\lambda'_A}$ (same for B) the unpolarized cross section would develop Boer-Mulders \otimes Collins effects ($h_1^{\perp} \otimes H_1^{\perp}$): negligible (checked numerically!).

[M. Anselmino, M. Boglione, U.D., E. Leader, S. Melis, F. Murgia, in preparation]

Single Spin Asymmetries: $A^\uparrow B \rightarrow \pi X$, $A_N = (d\sigma^\uparrow - d\sigma^\downarrow)/(d\sigma^\uparrow + d\sigma^\downarrow)$

- Configuration ($AB \equiv pp$):

AB center of mass frame - A along $+Z$ -axis, π in the XZ plane with \mathbf{p}_T parallel to $+X$ -axis. $S_A = \uparrow$ or \downarrow along Y -axis, $S_B = 0$.

- \mathbf{k}_\perp 's imply: non-planar $ab \rightarrow cd$ scattering; different \perp directions.

Parton hel. amplit.s in pp c.o.m in terms of hel. amplit.s in partonic c.o.m. frame.

1) Boost along $\mathbf{p}_a + \mathbf{p}_b$: $S \rightarrow S'$

2) Rotation to put \mathbf{p}'_a along Z axis: $S' \rightarrow S''$

$$\hat{M}_{\lambda_c, \lambda_d; \lambda_a, \lambda_b} = \hat{M}_{\lambda_c, \lambda_d; \lambda_a, \lambda_b}^0(\hat{s}, \hat{t}) e^{i \sum_m \lambda_m \xi_m} e^{i(\lambda_a - \lambda_b) \phi_c''}$$

where ξ_m are functions of \mathbf{k}_\perp 's: **phases in hard scattering**

\Rightarrow **Let's focus on Collins effect**

$$A_N = \int dPS[\Sigma(\uparrow, 0) - \Sigma(\downarrow, 0)]_C / \int dPS[\Sigma(\uparrow, 0) + \Sigma(\downarrow, 0)]$$

On summing over $\{\lambda\}$ [$\lambda_a \neq \lambda'_a$ and $\lambda_b = \lambda'_b$] we get ($b = q, \bar{q}, g$)

$$\begin{aligned}
 [\Sigma(\uparrow, 0) - \Sigma(\downarrow, 0)]_{qb} &= \left\{ \begin{aligned} &F_{+-}^{+-}(x_a, k_{\perp a}) \cos[\phi_a + \phi_c'' - \xi_a - \tilde{\xi}_a + \xi_c + \tilde{\xi}_c + \phi_{\pi}^H] \\ &- F_{-+}^{+-}(x_a, k_{\perp a}) \cos[\phi_a - \phi_c'' + \xi_a + \tilde{\xi}_a - \xi_c - \tilde{\xi}_c - \phi_{\pi}^H] \end{aligned} \right\} \\
 &\times \hat{f}_{b/B}(x_b, k_{\perp b}) \left[\hat{M}_1^0 \hat{M}_2^0(x_a, x_b, z; \mathbf{k}_{\perp a}, \mathbf{k}_{\perp b}, \mathbf{k}_{\perp \pi}) \right]_{qb} \\
 &\times [2 \text{Im} D_{+-}^{\pi}(z, k_{\perp \pi})]
 \end{aligned}$$

$$(\hat{M}_1^0)_{qb} \equiv \hat{M}_{+,+;+,+}^0 = \hat{M}_{-,-;-,-}^0 \quad (\hat{M}_2^0)_{qb} \equiv \hat{M}_{-,+;-,-}^0 = \hat{M}_{+,-;+,-}^0$$

The product $\hat{M}_1^0 \hat{M}_2^0$, is, for a collinear collision with scattering plane $\equiv XZ$, related to the spin transfer cross section:

$$\frac{1}{16\pi \hat{s}^2} \left[\hat{M}_1^0 \hat{M}_2^0 \right]_{qb} = \frac{d\hat{\sigma}^{q^\uparrow b \rightarrow q^\uparrow b}}{d\hat{t}} - \frac{d\hat{\sigma}^{q^\uparrow b \rightarrow q^\downarrow b}}{d\hat{t}}$$

Maximization of the Collins mechanism in $p^\uparrow p \rightarrow \pi X$

In the notations of P.J. Mulders, R.D. Tangerman *Nucl. Phys.* **B461** (1996):

$$F_{+-}^{+-}(x, k_\perp) = h_1(x, k_\perp)$$

$$F_{-+}^{+-}(x, k_\perp) = \frac{k_\perp^2}{2M_p^2} h_{1T}^\perp(x, k_\perp)$$

$$2 \text{Im} D_{+-}^\pi(z, k_\perp) = \Delta^N D_{\pi/q^\uparrow}(z, k_\perp) = \frac{2k_\perp}{zM_\pi} H_1^{\perp q}(z, k_\perp)$$

The following positivity bounds hold [A. Bacchetta, *et al. Phys. Rev. Lett.* **85** (2000)]:

$$|h_1(x, k_\perp)| \leq \frac{1}{2} [q(x, k_\perp) + \Delta q(x, k_\perp)] = q_+(x, k_\perp)$$

$$\frac{k_\perp^2}{2M_p^2} |h_{1T}^\perp(x, k_\perp)| \leq \frac{1}{2} [q(x, k_\perp) - \Delta q(x, k_\perp)] = q_-(x, k_\perp)$$

$$|\Delta^N D_{\pi/q^\uparrow}(z, k_\perp)| \leq 2D_{\pi/q}(z, k_\perp)$$

Let's consider $A_N(\pi^+)$: u, d active flavor for h_1 and h_{1T}^\perp :

$$h_1^u(x, k_\perp) = u_+(x, k_\perp) \quad h_1^d(x, k_\perp) = -d_+(x, k_\perp)$$

$$\frac{k_\perp^2}{2M_p^2} h_{1T}^{\perp u}(x, k_\perp) = -u_-(x, k_\perp) \quad \frac{k_\perp^2}{2M_p^2} h_{1T}^{\perp d}(x, k_\perp) = +d_+(x, k_\perp)$$

being the product of elementary amplitudes $\hat{M}_1^0 \hat{M}_2^0 < 0$, to get $A_N(\pi^+) > 0$

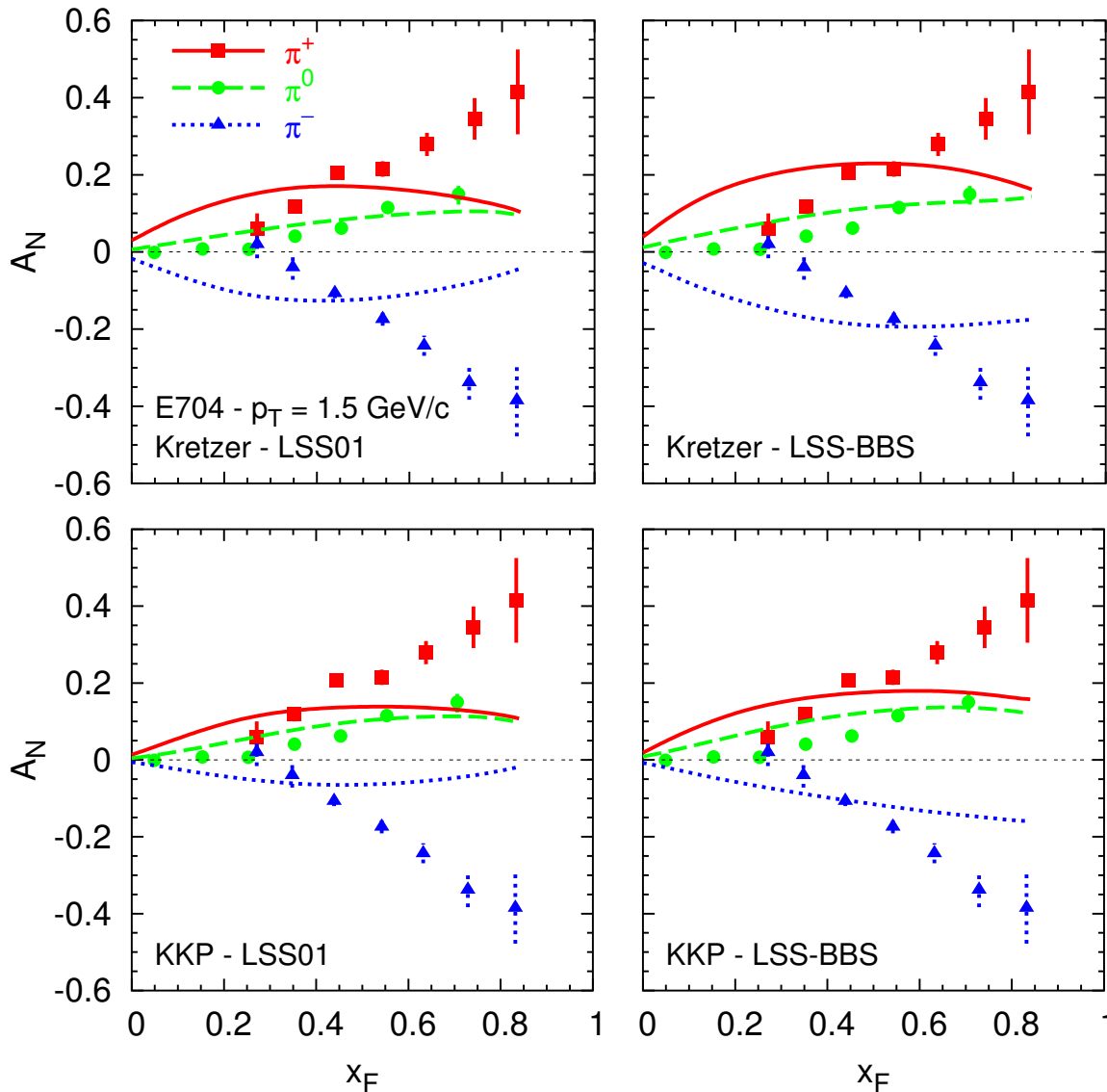
$$\Rightarrow \Delta^N D_{\pi^+/u^\uparrow}(z, k_\perp) = -2D_{\pi^+/u}(z, k_\perp)$$

Contribution (maximized) of the sub-leading channel *down* \rightarrow *down* (neglected in former studies); since $h_1^d < 0$, to add in sign, we use

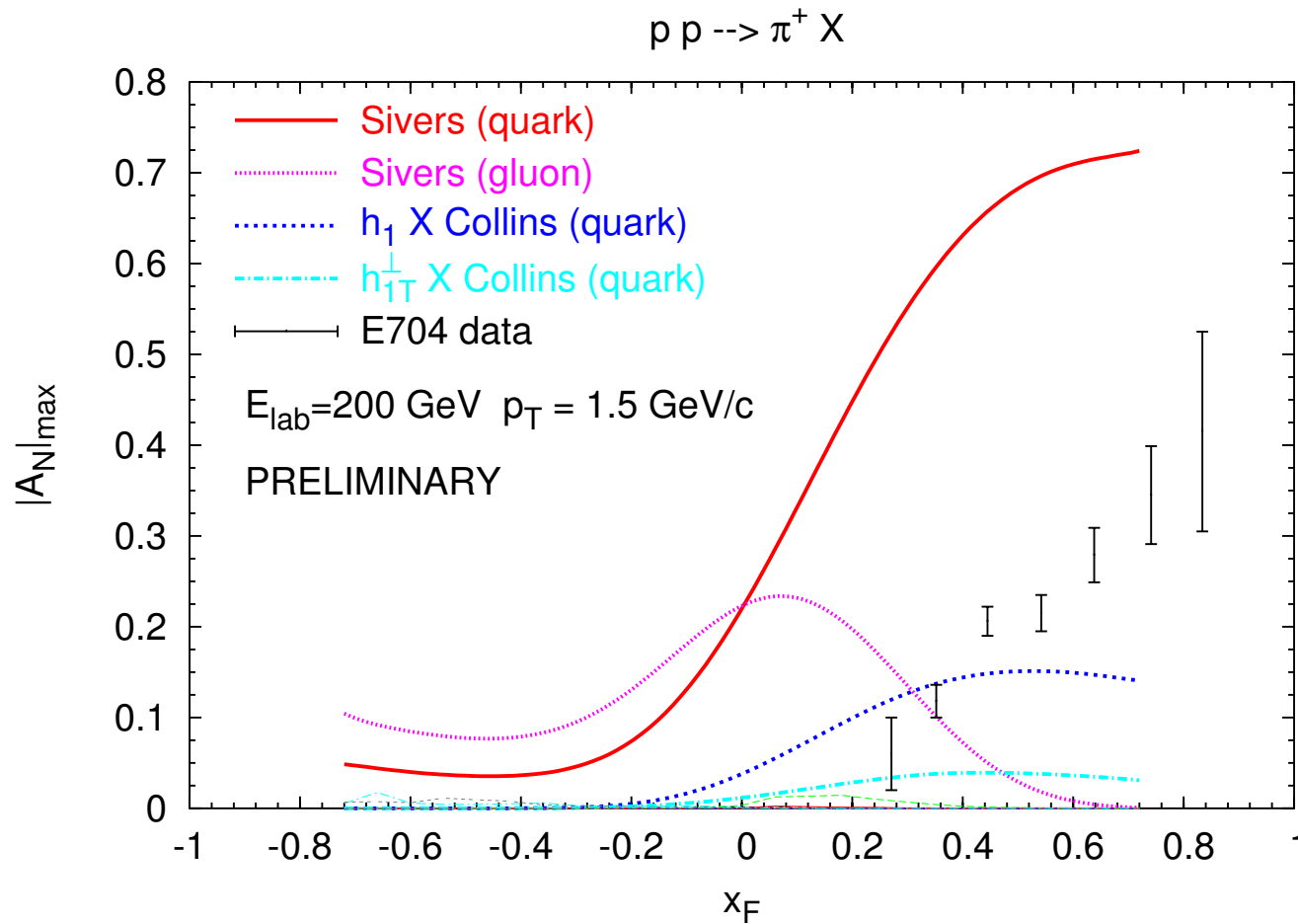
$$\Delta^N D_{\pi^+/d^\uparrow}(z, k_\perp) = +2D_{\pi^+/d}(z, k_\perp)$$

A_N for π^- 's is maximized in size (by isospin invariance).

- helicity PDF: LSS01 and LSS-BBS ($\Delta d > 0$ at large $x \Rightarrow$ bigger $|h_1^d|$)
- pion FF: Kretzer (sea $\simeq (1 - z)$ valence) and K KP (sea suppressed)



Maximized estimates
of $A_N(\pi)$ for
 $p^\uparrow p \rightarrow \pi X$ vs. x_F ,
at $\langle p_T \rangle = 1.5$ GeV/c
and $\sqrt{s} = 20$ GeV.
Collins mechanism
full saturated: all terms
added constructively.
Data are from [E704]
D.L. Adams *et al.*,
Phys. Lett. **B261** (1991)
and A. Bravar *et al.*,
Phys. Rev. Lett. **77** (1996).



**Maximized estimates of A_N , all possible contributions:
keeping only proper phases.**

Conclusions and outlook

- A generalized pQCD approach to SSA and unpol. cross sections:
LO-pQCD + spin and new \mathbf{k}_\perp -dependent PDF and FF;
- $A_N(p^\uparrow p \rightarrow \pi X)$: Complete \mathbf{k}_\perp -helicity formalism and phenomenology.
True detailed microscopic dynamics (proper phases) \Rightarrow
Sivers effect results **confirmed**;
Collins effect **suppressed** (against former claims!);
- $A_N(p^\uparrow p \rightarrow \pi X)$ at $x_F < 0$: strong suppression (of all mechanisms!!) by phase conspiracy;
- $p^\uparrow p \rightarrow \pi$ jet X : to extract Collins effect (less severe phase cancellations);
- $AB \rightarrow CX$ ($C = \pi, \Lambda, \rho$): Complete \mathbf{k}_\perp -helicity formalism and phenomenology for $A_{LL}, A_{TT}, A_{LT}, P$ (in progress). **Role of “T-odd” effects in $A_{LL}(\pi)$** ;
- Azimuthal spin asymmetries in SIDIS: role of Collins & Sivers effect (A. Prokudin talk + detailed \mathbf{k}_\perp approach in progress).