Higher twists resummation in inclusive and semi-inclusive spin-dependent DIS

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1. Spin in QCD: Leading and higher twists.
2. Resummation: where it is possible and mandatory?
3. Generalized GDH sum rules and BG duality: role of g_T
4. k_T−dependent functions; models for resummed twists.
5. Conclusions

Spin and higher twist

Free quarks density matrix

$\rho = \frac{1}{2}(\hat{p} + m)(1 + \hat{s}\gamma_5)$

At large energies mass is suppressed and longitudinal polarization is enhanced $S \to \xi p/m$ $\rho \to \frac{1}{2}\hat{p}(1 + \xi\gamma_5)$, Transversal polarization is described by two RELATED terms $\hat{s}\gamma_5\hat{p}$ and $\hat{s}\gamma_5 m$,

Parton distributions - quarks density matrix in nucleon: Parametrization of matrix elements

$$< P, S|\psi_\alpha(0)\hat{E}(0, z)\bar{\psi}_\beta(z)|P, S >= q(Pz)\hat{P} + M g_T(Pz)\hat{S}_T\gamma_5 - M g_2(Pz)\frac{(Sz)}{(Pz)}\hat{P}\gamma_5 + h_1(Pz)\hat{P}\hat{S}_T\gamma_5 + ...$$

Transverse spin - accompanied either by momentum (twist 2) or by mass parameter or COORDINATE (twist 3). Two SEPARATE (but constrained by Gauge Invariabce) descriptions of transverse spin.

Two sources of higher twists

Kinematical (Wandzura-Wilczek) due to

$z^2 \neq 0$ ($k_T \neq 0$)

Dynamical (genuine)

$G^{\mu\nu}(A_T^\mu) \neq 0$ -often small

Physical processes - mass dimension of higher twist compensated by some (large) scale. DIS - only Q^2 -contributions of order M^2/Q^2. Manifested at low Q^2.

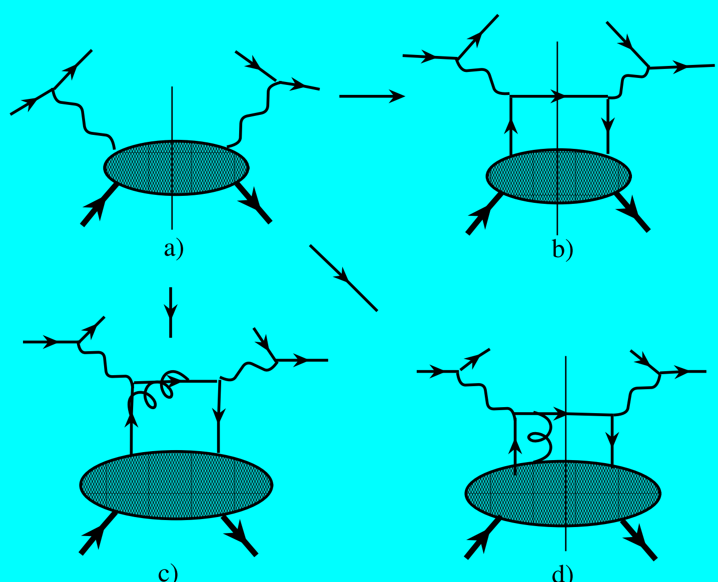
When ALL the twists are important? - Resonance region ($x \sim 1$). $M^2/Q^2 \to M^2/Q^2(1-x) \sim M^2/s$ (De Rujula, Georgi, Politzer)

Relation to parton language - Bloom-Gilman duality.

Spin-dependent case - how to apply?

Answer: proceed analogously to QCD sum rules: BG duality appears as a local quark-hadron duality.

1. Calculate (leading+higher twists) contribution to DIS



2. Write the (Borel) dispersion relation (with respect to $s = Q^2(1-x)$, which is a natural scale of higher twists)

3. Take the ansatz for spectral functions which includes RESONANCE contribution below the threshold defined by DUALITY interval and leading perturbative one above that threshold.

$$\rho(s,Q^2) = \theta(s-s_0)\rho^{Pert}(Q^2/(s+Q^2)) + \theta(s_0-s)\rho^{Res}(s,Q^2) \tag{1}$$

so that

$$\int_{s_{min}}^{s_0(M^2)} (\rho^{Res}(s) - \rho^{Pert}(s))e^{-s/M^2} ds = \sum_n a_n (\frac{M_h^2}{M^2})^n. \tag{2}$$

4. Put Borel parameter $M \to \infty$ (higher twists corrections disappear) and assume the finite limit of duality interval $\to$ BG duality.

$$\int_{s_{min}}^{s_0(\infty)} (\rho^{Res}(s) - \rho^{Pert}(s)) ds = 0. \tag{3}$$

Determination of the duality interval from QCD - requires the power corrections calculation.

However, immediate result - the BG duality appears for the TENSOR structures appearing at LO level only.

$$W_A^{\mu\nu} = \frac{-i\epsilon^{\mu\nu\alpha\beta}}{pq} q_\beta (g_1(x,Q^2)s_\alpha + g_2(x,Q^2)(s_\alpha - p_\alpha \frac{sq}{pq})) = \frac{-i\epsilon^{\mu\nu\alpha\beta}}{pq} q_\beta ((g_1(x,Q^2) + g_2(x,Q^2))s_\alpha - g_2(x,Q^2) p_\alpha \frac{sq}{pq}) \quad (4)$$

Only $g_T = g_1 + g_2$ appears at LO. BG should work for g_T, not g_2 or $g_1 = g_T - g_2$ Supported by data! $\Delta(1232)$, violating BG - contributes only to g_2.

Integrate over x - importance of higher twists - low Q^2 region - GGDH sum rule.

$$I_1(Q^2) = \frac{2M^2}{Q^2}\Gamma_1(Q^2) \equiv \frac{2M^2}{Q^2}\int_0^1 g_1(x,Q^2)dx \; . \qquad (5)$$

Large Q^2 - GGDH integral behaves like $1/Q^2$.
Lower Q^2 -$1/Q^4, 1/Q^6$....
BUT $Q^2 = 0$ (elastic contribution subtracted)

$$I_1(0) = -\frac{\mu_A^2}{4} \; , \qquad (6)$$

MODEL for resummed twists: $g_T = g_1 + g_2$ rather than $g_1 = g_T - g_2$ (J. Soffer, O.T.). Linear in μ_A - good candidate for NPQCD. Compatible with resonance approaches (Burkert, Ioffe): $\Delta(1232)$, providing the rapid change of $I_p(Q^2)$ - contributes only to g_2 -link with BG.

$$I_T(0) = +\frac{\mu_A}{4}\,, \tag{7}$$

may be smooth, while I_2 has sharp dependence from Burkhardt-Cottingham sum rule.

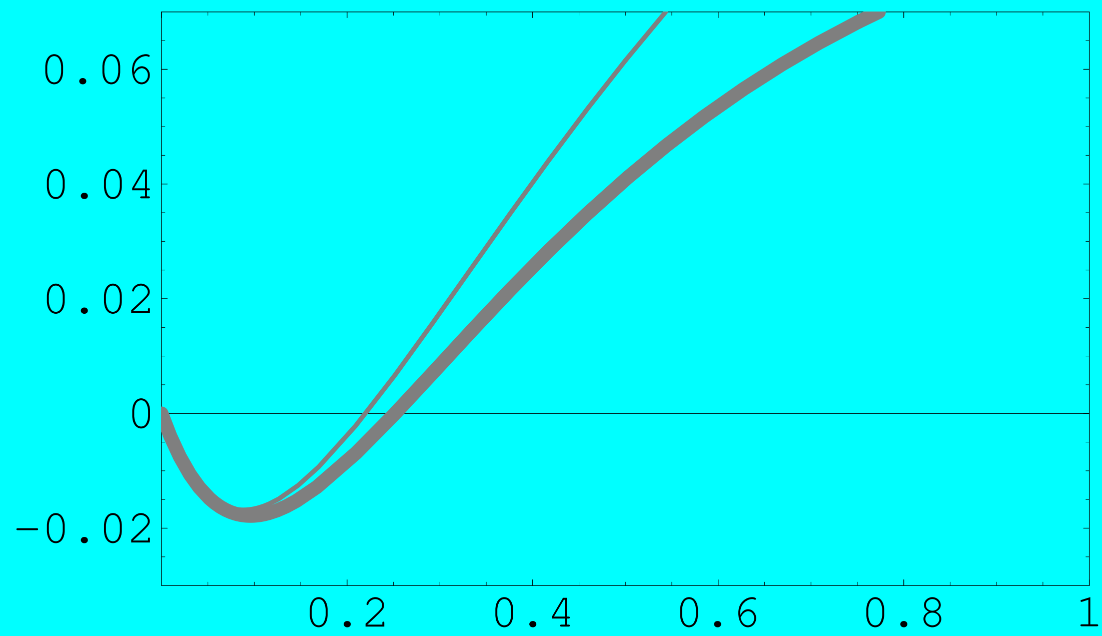
Inputs - leading twist, no radiative corrections (93),

or

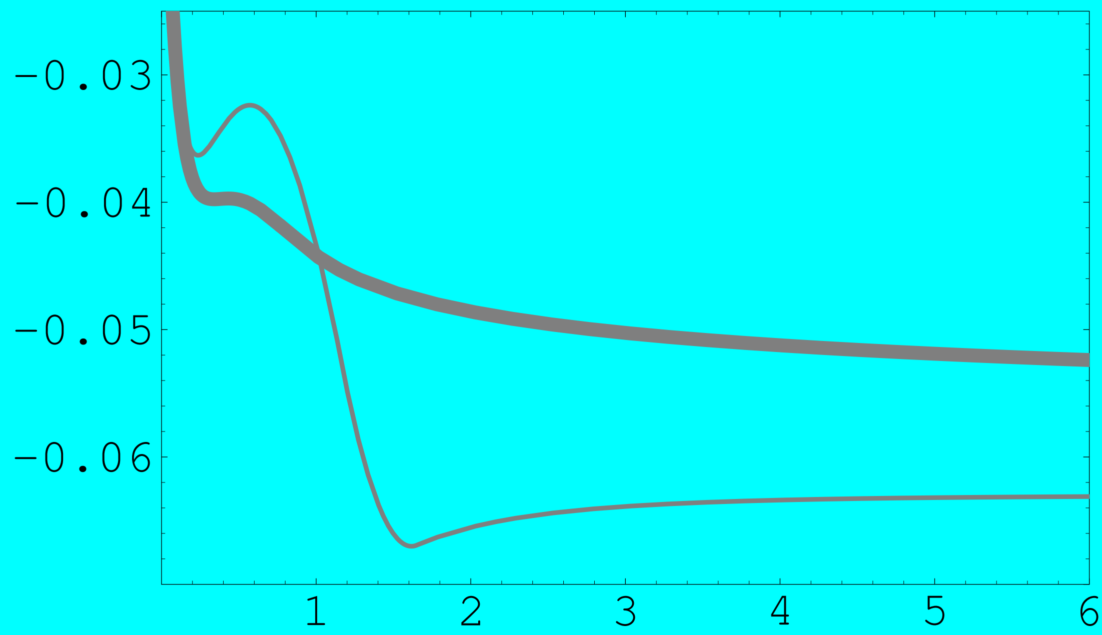
add subleading twist, 3-loops raidative corrections(04).

Matching of expansions in Q^2 ("chiral")and $1/Q^2$ ("twist") - may be justified by analyticity in Q^2.

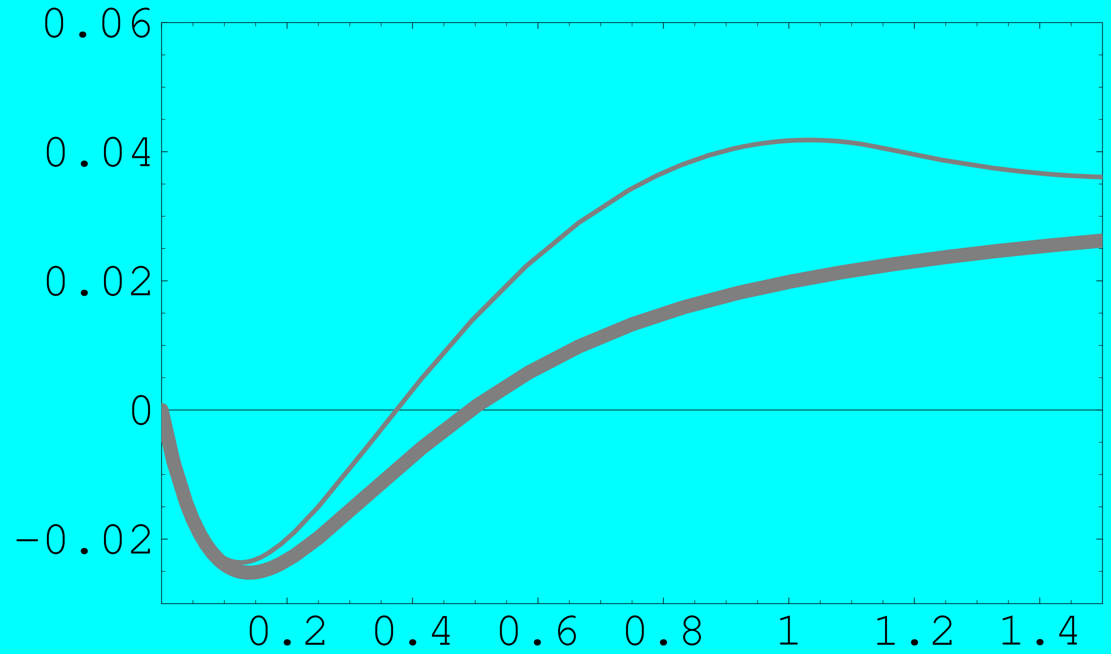
Results for proton



Neutron



Deuteron



Semi-inclusive processes (SIDIS , $pp \to HX$) various high twist contributions. The most important - SSA - M/p_T.

$\Delta\sigma \sim Mp_T$ kinematically - transverse polarization + parity conservation.

$A = \Delta\sigma/\sigma \sim Mp_T/p_T^2$ for large p_T.

For large and small P_T : $A \sim \frac{Mp_T}{p_T^2+M^2}$ - denominator from twist 5,7,... - result of resumation is restricted by symmetries. In the parton model - $sin\phi$ (A.V. Efremov (78)).

But- for small P_T - soft propagators - we cannot trust the answers.

Imaginary phase (T-odd effects in T-conserving theories) - loops including large and soft distances - genuine twist 3

Soft quarks - fermionic poles (A.V. Efremov, O.T. (85,95)) (DY - no suppression as $1/Q$, still behaves like M/p_T). But consideration of small $P_T \sim M$ - illegitimate - all twists are important.

Soft gluons - gluonic poles - considered to be dominant for large x_F(easier to emit soft gluon than soft quark) - detailed numerics exists (J.Qiu, G. Sterman (91,98))

However

Gluonic poles predictions for large X_F - go above RHIC data.

Gluonic poles - related to genuine twist contribution to $\int dx x^2 g_2$ - small.

Fermionic poles should be reanalyzed.

Another argument - from unpolarized cross-sections

NLO - much better description of RHIC than E704 kinematics (C. Bourrely, J. Soffer (2003); plenary talk ofW. Vogelsang: role of resummations).

Why (no explicit energy dependence in QCD) ?

Possible origin - HT - the typical partonic $x_i \sim p_T/\sqrt{s}$ - larger for E704. HT typically grow with x - less important for RHIC. Also k_T dependence (talks of E. Boglione, U. d'Alesio, F. Murgia) - model for high twist, More important for E704.

Gluonic poles: $\Delta\sigma \sim 1/(1-x)$ - to preserve positivity requires the growing with x spin independent HT term. If it is small, gluonic poles also cannot be large.

Another source of P_T and SSA - $k_T - dependent$ distribution (fragmentation) functions.

What is their twist?

Collins function $< \bar{q}\sigma^{\mu\nu}q >\sim M^{-1}H(z,k_T)\epsilon_{\mu\nu P k_T}$. M in denominator! As $k_T \sim M$ no suppression - leading twist.

Definition in coordinate space (O.T. (03))

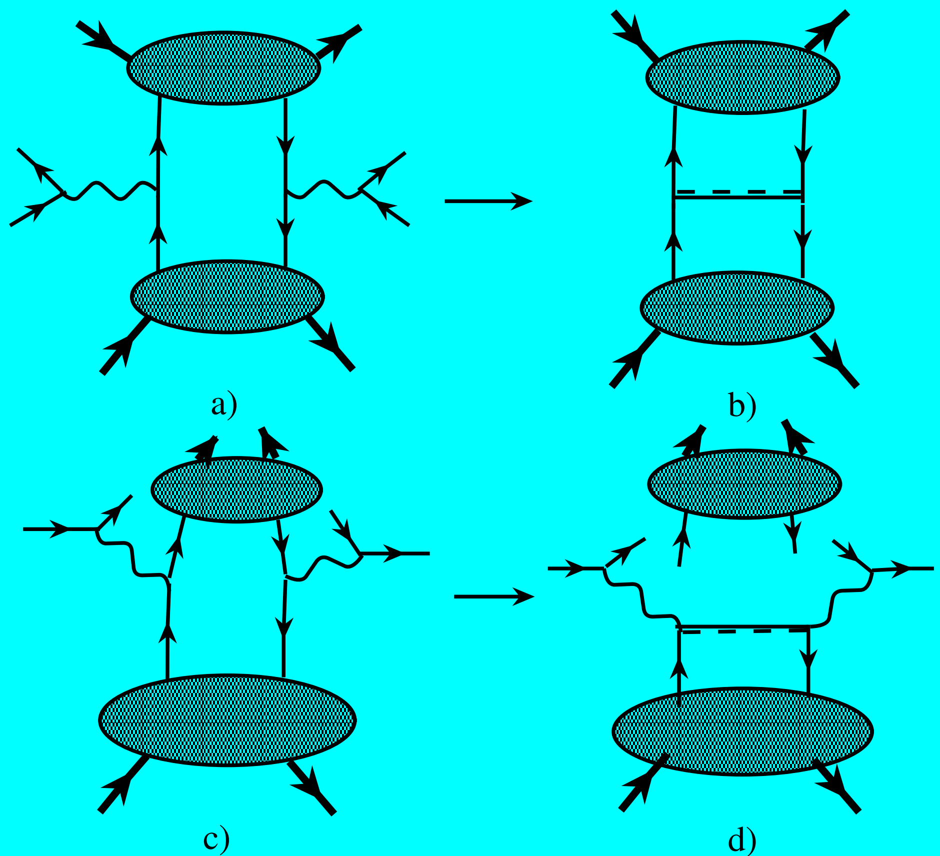
$< \bar{q}(0)\sigma^{\mu\nu}q(z) >\sim MI(z)\epsilon_{\mu\nu P z}, z \cdot A(z) = 0$

M in numerator - normal twist 3 (like g_2).

Helps in factorization proof. Complementary to Ji et al.

Problems in factorization at low p_T - no hard subprocess at Born level.

Solved long ago in DY (Efremov, Radyushkin (80))- integration over transverse momentum provide, due to geberalized optical theorem the effective "propagator". SIDIS -analogously (O.T. (03))



DY

$$\bar{W}^{\mu\nu}(M^2, X_F) =$$

$$\int d^4q\delta(q^2 - M^2)\delta(\frac{2q\cdot(p_1 - p_2)}{s} - x_F)W^{\mu\nu}(p_1, p_2, q)$$

$$= Disc_s \int \frac{d^4q}{2\pi(q^2 - M^2)} W^{\mu\nu}(p_1, p_2, q)\delta(\frac{2q\cdot(p_2 - p_1)}{s} - x_F)$$

SIDIS

$$\bar{W}^{\mu\nu}(q^2, x_B, z) = \int d^4p_3\delta(p_3^2)\delta(\frac{p_1p_3}{p_1q} - z)W^{\mu\nu}(p_1, p_3, q) \rightarrow$$

$$Disc_s \int \frac{d^4p_3}{2\pi p_3^2} W^{\mu\nu}(p_1, p_3, q)\delta(\frac{p_1p_3}{p_1q} - z)$$

For DY - heavy propagator - independent distributions for colliding hadrons. SIDIS - only after the assumption on independent (or common - fracture function) non-perturbative inputs for initial and final hadron factorization may be justified.

Spin-dependent SIDIS - weighted integration over p_T: (Kotzinian and Mulders

$$\Delta_n \bar{W}^{\mu\nu}(q^2, x_B, z) = \int d^4 p_3 \delta(p_3^2)(p_3 n)\delta(\frac{p_1 p_3}{p_1 q} - z)\Delta W^{\mu\nu}(p_1, p_3, q), \quad (8)$$

where n is the reference transverse vector - well defined EM GI result

$$\Delta_n \bar{W}_n^{\mu\nu}(q^2, x_B, z) = \frac{M x_B h(x_B) z^3 I(z)}{Q^2}(x_B p_1^{[\mu} \epsilon^{\nu]nSp_1} + p_1^{\mu} \epsilon^{\nu Sqn} + q^{\nu} \epsilon^{\mu n S p_1} - S^{\mu} \epsilon^{\nu Pqn} - n^{\nu} \epsilon^{\mu q S p_1}) \quad (9)$$

$\vec{n}$ - plays the role of $\vec{k}_T$ direction in calculation with standard Collins function: $Tr[\hat{p}_1 \hat{S} \gamma_5 \gamma^{\mu} \hat{p}_3 \hat{n} \gamma^{\nu}] \to [\hat{p}_1 \hat{S} \gamma_5 \gamma^{\mu} \hat{p}_3 \hat{k}_T \gamma^{\nu}]$. This does not change the azimuthal asymmetry as $< d\sigma(\vec{p}_3 \vec{n}) > \to$ $< d\sigma(\phi_h) cos(\phi_h - \phi_n) >= cos\phi_n < d\sigma(\phi_h) cos(\phi_h) > + sin\phi_n$ $< d\sigma(\phi_h) sin(\phi_h) >$. However : Attempt to fix p_T - singular cross-section (suggests the correct experimental procedure).

Correspondence to the Collins function

$$I(z) \sim \int dk_T^2 \frac{k_T^2}{M^2} H_1(z, k_T^2), \quad (10)$$

M^2 -because in the coordinate space M is in numerator and in the momentum space - in the denominator. Trace of twist - 3 nature: $\frac{\int d^2 p_T |p_T| ...}{\int d^2 p_T ...} \sim M$ - not $|Q|$. Here we deal with M/p_T effects with $p_T \sim M$ - contrary to twist subprocess - legitimate - no soft propagators

Higher moments in P_T (for simplicity unpolarized DY): $(P_T)^{2n} \sim M^{2n}$:

$$\bar{W}^{\mu\nu m}(M^2, x_F) = \int d^4q\delta(q^2 - M^2)W^{\mu\nu}(p_1, p_2, q)(qn)^{2m}$$
$$\sim M^{2m} \sum_k a_1^k a_2^{m-k} \quad (11)$$

where a are the Taylor coefficients of twist-resummed quark correlators in hadrons - Not seen in DIS! Determine the p_T shape absent in inclusive processes. Hadronic analogs of non-local quark condensates (Gromes (84); Mikhailov, Radyushkin (85)), Ansatz for higher twists RESUMMATION, Specific problem - analytic continuation to pseudo-Euclidian space.

Coordinate space counterparts of k_T-dependent distributions

$$\hat{q}_i(z) = \hat{p}_i \int_0^1 dy e^{ix_i py} q_i(x_i, M^2 z^2) =$$
$$\sum_n a_n (M^2 z^2)^n \int_0^1 dy e^{ix_i py} q_i(x_i), \quad (12)$$

The finiteness of all moments - requires cross-section to decrease faster than any power of P_T- natural explanation of exponential dependence. Intuitive picture of TMD persists.

Non-local quark condensates - eliminate the non-physical singularities $\sim \delta(x), \delta(1-x)$ of pion light cone DA $\phi(x)$ in favour of peaks of finite width.

Resummed quark correlators - eliminate the non-physical $\delta(k_T^2)$ dependence of the cross-sections in favour of finite width.

TMD - analog of quark-hadron duality.

Meson spectrum - real Breit-Wigner form cannot be deduced from QCD - requires infinitely many condensates.

However - averaged quantities (masses, couplings) are well described.

TMD - averaged description of k_T-shape may be justified. The full form requires twist resummation procedure.

CONCLUSIONS

Higher twists resummation in the absence of rigorous reulsts - ansatz problem.

1. Bloom -Gilman duality - analogous to local quark hadron duality in QCD SR framework Should hold for g_T not g_1, g_2. Combined effect of higher twists - duality interval.

2. GGDH sum rule - g_T - natural candidate for HT resummation - analytical ansatz.

3. Coordinate analogs of TMD functions

- allow for the proof of factorization in SIDIS

-reveal the twist 3 nature of Collins function

-establish the in-hadron analogs to the non-local quark condensates giving the quantitative sence to the understanding of TMD as a higher twist resummation.

-provide the natural explanation of exponential falloff of cross-sections with P_T