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Large distance effects on spin observables at RHIC

* Introduction

* Hadron elastic scattering at HE
  and effect of CNI

* Spin effects in the different approximations

* Slope of the spin-flip amplitude

* The case of the proton-nuclei scattering

* The form of A_N and energy dependence
  of the spin-flip amplitude.

* The impact of the saturaion on A_N

* Summary

If we define the forms of the separate hadron scattering amplitude as:

$$\mathbf{Im}\ \mathcal{A}_{\mathbf{nf}}(\mathbf{s},\mathbf{t}) \ \sim \ \mathbf{exp}(\mathbf{B_1^+}\ \mathbf{t}), \quad \mathbf{Re}\ \mathcal{A}_{\mathbf{nf}}(\mathbf{s},\mathbf{t}) \sim \mathbf{exp}(\mathbf{B_2^+}\ \mathbf{t}),$$
$$\mathbf{Im}\tilde{\mathcal{A}}_{\mathbf{sf}}(\mathbf{s},\mathbf{t}) \ \sim \ \mathbf{exp}(\mathbf{B_1^-}\ \mathbf{t}), \quad \mathbf{Re}\tilde{\mathcal{A}}_{\mathbf{sf}}(\mathbf{s},\mathbf{t}) \ \sim \ \mathbf{exp}(\mathbf{B_2^-}\ \mathbf{t}),$$

then, at small t ($\sim 0 \div 0.1\ GeV^2$), practically all semiphenomenological analyses assume:

$$B_1^+ \ \approx \ B_2^+ \ \approx \ B_1^- \ \approx \ B_2^-.$$

Actually, we can take the eikonal representation for the scattering amplitude

$$\phi_{\mathbf{1}}^{\mathbf{h}}(\mathbf{s},\mathbf{t}) \ = \ -\mathbf{ip}\int_{\mathbf{0}}^{\infty} \ \rho\ \mathbf{d}\rho\ \mathbf{J_0}(\rho\mathbf{q})\ [\mathbf{e}^{\chi_{\mathbf{0}}(\mathbf{s},\rho)} \ - \ \mathbf{1}],$$
$$\phi_{\mathbf{5}}^{\mathbf{h}}(\mathbf{s},\mathbf{t}) \ = \ -\mathbf{ip}\int_{\mathbf{0}}^{\infty} \ \rho\ \mathbf{d}\rho\ \mathbf{J_1}(\rho\mathbf{q})\ \chi_{\mathbf{1}}(\mathbf{s},\rho)\ \mathbf{e}^{\chi_{\mathbf{0}}(\mathbf{s},\rho)}.$$

where $\chi_{\mathbf{0}}(\mathbf{s},\rho)$ represents the corresponding interaction potential $V_i(\rho,z)$ in impact parameter space.

$$\chi_{\mathbf{0}}(\mathbf{s},\rho) = \frac{\mathbf{1}}{\mathbf{2ip}}\int_{-\infty}^{\infty}\mathbf{dz}\ \mathbf{V_0}(\vec{\rho},\mathbf{z});$$

$$\chi_{\mathbf{1}}(\mathbf{s},\rho) = \frac{\rho}{\mathbf{2}}\int_{-\infty}^{\infty}\mathbf{dz}\ \mathbf{V_1}(\vec{\rho},\mathbf{z})$$

If the potentials V_0 and V_1 are assumed to have a Gaussian form in the first Born approximation, ϕ_1^h and $\hat{\phi}^h{}_5$ will have the same Gaussian form

$$\phi_1^h(s,t) \sim \int_0^\infty \rho\, d\rho\, J_0(\rho q)\, e^{-\rho^2/2R^2} = R^2\, e^{R^2 t/2},$$
$$\phi_5^h(s,t) \sim \int_0^\infty \rho^2\, d\rho\, J_1(\rho q)\;\; e^{-\rho^2/2R^2} = q\;\; R^4\, e^{-R^2 t/2}.$$

In this special case the slopes of the spin-flip and "residual" spin-non-flip amplitudes are indeed the same.

A Gaussian form for the potential is adequate to represent the central part of the hadronic interaction. This form cuts off the Bessel function and the contributions at large distances.

$$J_0(x) \simeq 1 - (x/2)^2; \quad \text{and} \quad 2\, J_1/x = (1 - 0.5\,(x/2)^2),$$

the corresponding integrals

$$\phi_1^h(s,t) \sim \int_0^\infty \rho\, d\rho\, (1 - \rho^2 \frac{q^2}{4})\, e^{-\rho^2/2R^2} \approx R^2 e^{-R^2\; q^2/2},$$
$$\phi_5^h(s,t) \sim \int_0^\infty \rho^2\, d\rho\, \rho\, \frac{q}{2}\, (1 - \rho^2 \frac{q^2}{8})\;\; e^{-\rho^2/2R^2} \approx q\;\; R^4\, e^{-R^2 q^2/2}.$$

have the same behavior in q^2

Let us examine the contribution of the large distances. The Hankel asymptotic of the Bessel functions at large distances are

$$
\begin{aligned}
\mathbf{J}_\nu(\mathbf{z}) &= \sqrt{\mathbf{2}/\pi\mathbf{z}}\ [\mathbf{P}(\nu,\mathbf{z})\ \cos\chi(\nu,\mathbf{z})\ -\ \mathbf{Q}(\nu,\mathbf{z})\ \sin\chi(\nu,\mathbf{z})],\\
\mathbf{P}(\nu,\mathbf{z}) &\sim \sum_{\mathbf{k}=\mathbf{0}}^{\infty}(-\mathbf{1})^{\mathbf{k}}\ \frac{(\nu,\ \mathbf{2k})}{(\mathbf{2z})^{\mathbf{2k}}},\\
\mathbf{Q}(\nu,\mathbf{z}) &\sim \sum_{\mathbf{k}=\mathbf{0}}^{\infty}(-\mathbf{1})^{\mathbf{k}}\ \frac{(\nu,\ \mathbf{2k}+\mathbf{1})}{(\mathbf{2z})^{\mathbf{2k}+\mathbf{1}}},\\
\chi(\nu,\mathbf{z}) &= z-(\nu/2+1/4)
\end{aligned}
$$

That gives for the $\nu = 1, 2$

$$
\begin{aligned}
\mathbf{J}_\mathbf{0}(\mathbf{x}) &= \sqrt{\mathbf{2}/(\pi\mathbf{x})}[\mathbf{P}_\mathbf{0}(\mathbf{x})\cos(\mathbf{x}-\pi/\mathbf{4})-\mathbf{Q}_\mathbf{0}(\mathbf{x}),\sin(\mathbf{x}-\pi/\mathbf{4})]\\
\mathbf{J}_\mathbf{1}(\mathbf{x}) &= \sqrt{\mathbf{2}/(\pi\mathbf{x})}[\mathbf{P}_\mathbf{1}(\mathbf{x})\cos(\mathbf{x}-\mathbf{3}\pi/\mathbf{4})-\mathbf{Q}_\mathbf{1}(\mathbf{x})\sin(\mathbf{x}-\mathbf{3}\pi/\mathbf{4})],
\end{aligned}
$$

with

$$
\begin{aligned}
\mathbf{P}_\mathbf{0}(\mathbf{x}) &\approx \mathbf{1}-\mathbf{0.0703125}/\mathbf{x}^\mathbf{2}+\mathbf{0.1121521}/\mathbf{x}^\mathbf{4}+...;\\
\mathbf{Q}_\mathbf{0}(\mathbf{x}) &\approx -\mathbf{0.125}/\mathbf{x}+\mathbf{0.073242188}/\mathbf{x}^\mathbf{3}+...;\\
\mathbf{P}_\mathbf{1}(\mathbf{x}) &\approx \mathbf{1}+\mathbf{0.1171875}/\mathbf{x}^\mathbf{2}-\mathbf{0.144195557}/\mathbf{x}^\mathbf{4}+...;\\
\mathbf{Q}_\mathbf{1}(\mathbf{x}) &\approx \mathbf{0.375}/\mathbf{x}-\mathbf{0.10253906}/\mathbf{x}^\mathbf{3}+....
\end{aligned}
$$

From this, we obtain:

$$
\begin{aligned}
\sqrt{\pi\mathbf{x}}\mathbf{J}_\mathbf{0}(\mathbf{x}) &\approx [(\mathbf{1}-\frac{\mathbf{0.125}}{\mathbf{x}}-\frac{\mathbf{0.07}}{\mathbf{x}^\mathbf{2}})\cos\mathbf{x}+(\mathbf{1}+\frac{\mathbf{0.125}}{\mathbf{x}}-\frac{\mathbf{0.07}}{\mathbf{x}^\mathbf{2}})\sin\mathbf{x};\\
\sqrt{\pi\mathbf{x}}\mathbf{J}_\mathbf{1}(\mathbf{x}) &\approx [(\mathbf{1}+\frac{\mathbf{0.375}}{\mathbf{x}}+\frac{\mathbf{0.117}}{\mathbf{x}^\mathbf{2}})\sin\mathbf{x}-(\mathbf{1}-\frac{\mathbf{0.375}}{\mathbf{x}}+\frac{\mathbf{0.117}}{\mathbf{x}^\mathbf{2}})\cos\mathbf{x}].
\end{aligned}
$$

Let us calculate the corresponding integrals in the case of large distances

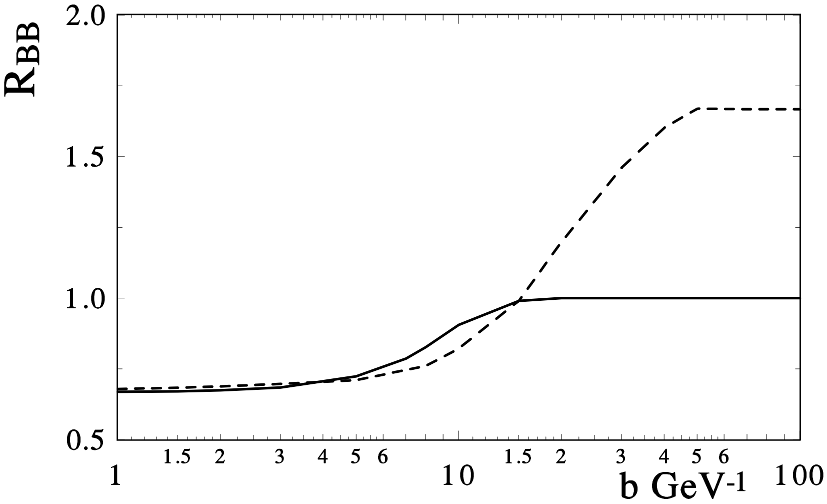
$$
\begin{aligned}
\phi_1^h(s,t) &\sim \int_1^\infty \rho^2/\sqrt{q\ \rho}\ \ \exp[-\rho^2/(2R^2)]\ d\rho \\
&= 1/\sqrt{2\ q}\ \ R^{3/2}\ \gamma[3/4;1/(2R^2)] \\
\phi_5^h(s,t)/q &\sim 1/q \int_0^\infty \rho^2/\sqrt{q\ \rho}\ \ \exp[-\rho^2/(2R^2)]\ d\rho \\
&= 1/(q\ \sqrt{q})\ \ 2^{1/4}\ \ R^{5/2}\ \ \gamma[5/4;1/(2R^2)].
\end{aligned}
$$

where $\gamma[a,z]$ is the incomplete Gamma function.

The exponential asymptotics of both representations are the same, but the reduced spin-flip amplitude has an additional $q^{3/2}$ in the denominator and hence it has a larger effectively slope then the spin non-flip amplitude.

$$
\begin{aligned}
\phi_1^h(s,t) \sim 1/q^2 \int_0^\infty \sqrt{x} &\quad [(1-0.125/x)\cos x \\
+(1+0.125/x)\sin x]e^{-x^2/(2R^2q^2}\ dx &\quad \approx R/q\ {}_1F_1(3/4,1/2,-q^2R^2/2), \\
\frac{\phi_5^h(s,t)}{q} \sim \frac{1}{q^4}\int_0^\infty\ x^{3/2}\ [(0.375/x-1) &\quad \cos x + (1-\frac{0.375}{x})\sin x]e^{-x^2/2Rq^2}dx \\
\approx R^{3/2}/q^{5/2} &\quad {}_1F_1(3/4,1/2,-q^2R^2/2),
\end{aligned}
$$

Again the additional $q^{3/2}$ in the denominator leads to a larger slope for the residual spin-flip-amplitude.



The cases of exponential and Gaussian impact parameter representation as function of the upper limit b of the corresponding integral

$$\phi_1^h(t) \sim \int_0^b \rho \; d\rho \; J_0(\rho\Delta) f_n,$$

$$\phi_5^h(t)/q \sim \int_0^b \rho^2 \; d\rho \; J_1(\rho\Delta) \; f_n.$$

with $f_n = \exp\left[-(\rho/5)^n\right]$, and $n = 1, 2$

We calculate the ratio of the slopes of these two amplitudes

$$R_{BB} = B^{nf}/B^{sf}$$

as a function b for these two values of n.

In the case of an exponential tail for the potentials

$$\chi_{\mathbf{i}}(\mathbf{b},\mathbf{s}) \ \sim \ \mathbf{H}\ \mathbf{e}^{-\mathbf{a}\ \rho},$$

and using the standard integral representation

$$\int_0^\infty \mathbf{x}^{\alpha-1}\ \exp(-\mathbf{p}\ \mathbf{x})\mathbf{J}_\nu(\mathbf{c}\mathbf{x})\ \mathbf{d}\mathbf{x} \ = \ \mathbf{I}_\nu^\alpha,$$

with

$$\mathbf{I}_\nu^{\nu+2} \ = \ \mathbf{2p}\ (\mathbf{2c})^\nu\ \Gamma(\nu+\mathbf{3/2})\mathbf{1}/[\sqrt{\pi}(\mathbf{p}^2+\mathbf{c}^2)^{3/2}],$$

one obtains

$$\int \rho\ \mathbf{d}\rho\ \mathbf{e}^{-\mathbf{a}\ \rho}\ \mathbf{J}_0(\rho\mathbf{q}) = \frac{\mathbf{a}}{(\mathbf{a}^2+\mathbf{q}^2)^{3/2}} \approx \frac{\mathbf{1}}{\mathbf{a}\sqrt{\mathbf{a}^2+\mathbf{q}^2}}\ \mathbf{e}^{-\mathbf{B}\mathbf{q}^2}$$

with $\mathbf{B} \ = \ 1/\mathbf{a}^2$, and for small x

$$1/(1+x) \ \sim \ (1-x) \ \sim \ exp(-x).$$

For the "residual" spin-flip amplitude

$$\begin{aligned}\sqrt{|\mathbf{t}|}\tilde{\mathbf{F}}_{\mathbf{sf}}(\mathbf{s},\mathbf{t}) \ &= \ \int \rho^2\ \mathbf{d}\rho\ \mathbf{e}^{-\mathbf{a}\ \rho}\ \mathbf{J}_1(\rho\mathbf{q}) \\ &= \ \frac{\mathbf{3\ a\ q}}{(\mathbf{a}^2+\mathbf{q}^2)^{5/2}} \approx \frac{\mathbf{3\ aq\ B}^2}{\sqrt{\mathbf{a}^2+\mathbf{q}^2}}\ \ \mathbf{e}^{-2\ \mathbf{B}\mathbf{q}^2}\end{aligned}$$

If, further, we take the first Born term of an inverse power form factor

$$\chi(\mathbf{s}, \rho) \;=\; \int \mathbf{q}\ \mathbf{dq}\ \mathbf{J_0}(\rho \mathbf{q}) \frac{\mathbf{\Lambda^{2n}}}{(\mathbf{\Lambda^2} + \mathbf{q^2})^{\mathbf{n}}} \;=\; \frac{\mathbf{\Lambda^{2n}}}{\mathbf{48}} (\mathbf{\Lambda}\rho)^{\mathbf{n-1}} \mathbf{K_{n-1}}(\mathbf{\Lambda}\rho),$$

the spin-flip amplitude is given by

$$\begin{aligned}
\sqrt{|\mathbf{t}|}\tilde{\mathbf{F}}_{\mathbf{sf}}(\mathbf{s}, \mathbf{t}) &= \frac{\mathbf{\Lambda^2}}{\mathbf{48}} \int \rho^2\ \mathbf{d}\rho\ \mathbf{J_1}(\rho \mathbf{q}) \mathbf{K_{n-1}}(\mathbf{\Lambda}\rho) \\
&= \frac{\mathbf{8\ q}}{\mathbf{\Lambda^2}} \ \ \frac{\mathbf{1}}{(\mathbf{1} + \mathbf{q^2}/\mathbf{\Lambda^2})^{\mathbf{n+1}}}
\end{aligned}$$

The derivative relations for the helicity amplitudes with $t \neq 0$ for spin-dependent amplitudes, carefully examined by B. Schrempp and F.Schrempp, give

$$\mathbf{F_{\lambda_i}(s,t) = C_{\lambda_i}(s)\sqrt{-t}^{\Delta\lambda}\left(\frac{1}{\sqrt{-t}}\frac{\partial}{\partial\sqrt{-t}}\right)^{\Delta\lambda} F_{\Delta\lambda=0}(s,t)}$$

If we take the spin non-flip amplitude in a non-Gaussian form, for example, as proportional to the form factor of hadrons

$$\mathbf{\Lambda^2/(\Lambda^2 - t)^n}$$

the spin-flip amplitude is given by

$$\mathbf{F_{1/2,-1/2}(s,t) = C_{1/2,-1/2}(s)\sqrt{-t}\frac{\Lambda^2}{(\Lambda^2 - t)^{n+1}}}.$$

Take the hadron spin non-flip amplitude in a simple exponential form:

$$\mathcal{A}^{\mathbf{pA}}_{\mathbf{nf}}(\mathbf{s},\mathbf{t}) \ = \ (1+\rho^{pA})\frac{\sigma^{\mathbf{pA}}_{\mathbf{tot}}(\mathbf{s})}{\mathbf{4}\pi}\exp\left(\frac{\mathbf{B}^{+}}{\mathbf{2}}\mathbf{t}\right).$$

We parametrize the spin-flip part of $p^{12}C$ scattering as

$$\mathcal{A}^{\mathbf{h}}_{\mathbf{sf}}(\mathbf{s},\mathbf{t}) \ = \ (\mathbf{k_2} \ + \ \mathbf{i} \ \mathbf{k_1})\frac{\sqrt{|\mathbf{t}|} \ \sigma^{\mathbf{pA}}_{\mathbf{tot}}(\mathbf{s})}{\mathbf{4}\pi}\exp\left(\frac{\mathbf{B}^{-}}{\mathbf{2}}\mathbf{t}\right),$$

In our analysis the elastic and total cross sections and the analysing power $\mathbf{A_N}$ for $p^{12}C$ scattering are given by

$$\mathbf{d}\sigma/\mathbf{dt} \ = \ \mathbf{2}\pi\left(|\mathcal{A}_{\mathbf{nf}}|^{\mathbf{2}}+|\mathcal{A}_{\mathbf{sf}}|^{\mathbf{2}}\right),$$

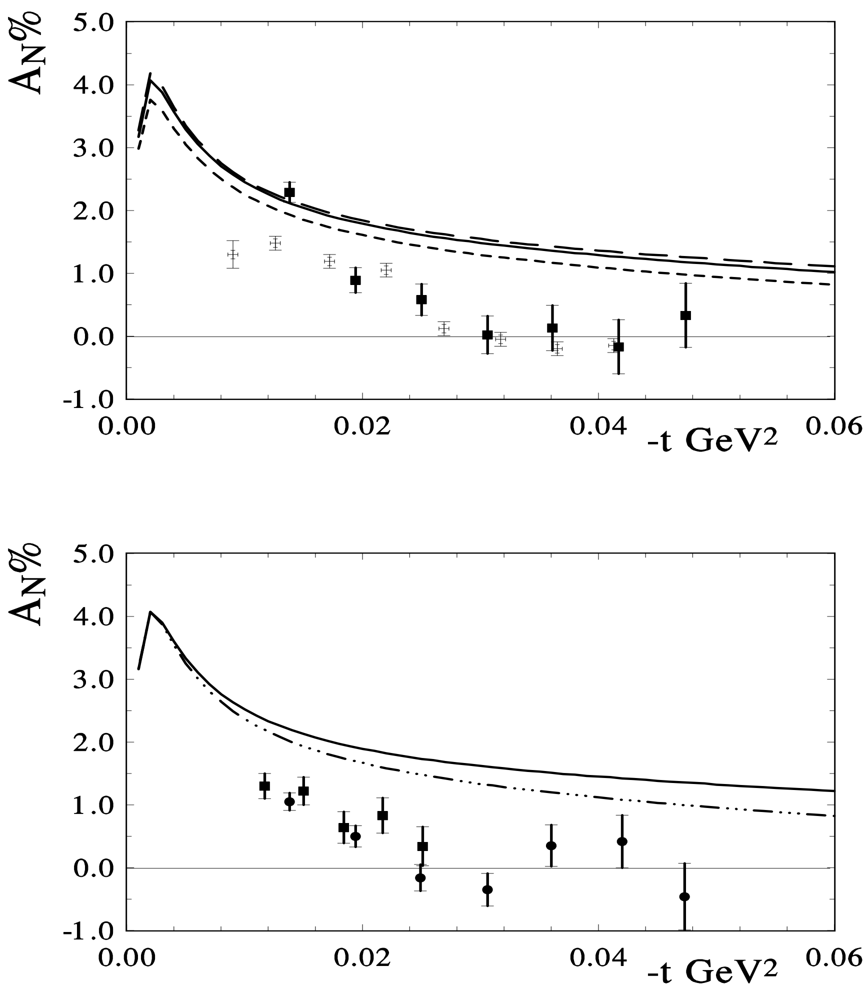
$$\sigma_{\mathbf{tot}} = \mathbf{4}\pi\mathbf{Im}(\mathcal{A}_{\mathbf{nf}}),$$

$$\mathbf{A_N} \ \mathbf{d}\sigma/\mathbf{dt} \ = \ -\mathbf{2}\pi\mathbf{Im}[\mathcal{A}_{\mathbf{nf}}\mathcal{A}^{*}_{\mathbf{sf}})].$$

From the full scattering amplitude, the analyzing power is given by

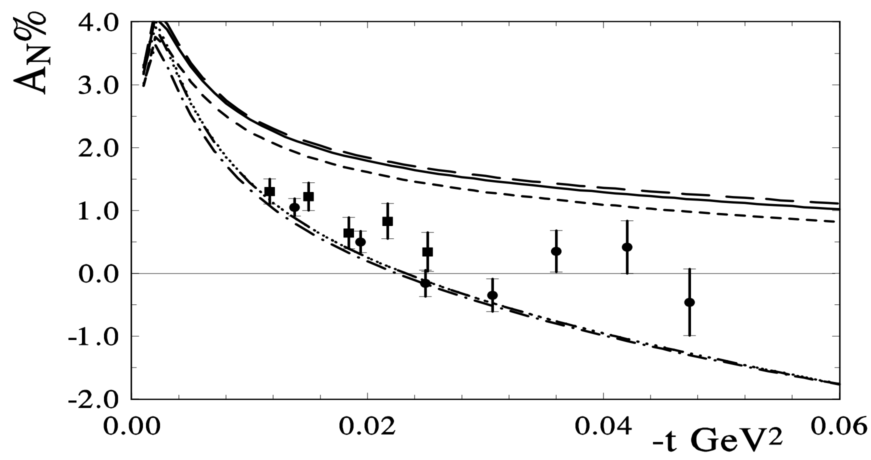
$$\mathbf{A_N}\frac{\mathbf{d}\sigma}{\mathbf{dt}} = -\mathbf{4}\pi[\mathbf{Im}(\mathcal{A}_{\mathbf{nf}})\mathbf{Re}(\mathcal{A}_{\mathbf{sf}}) - \mathbf{Re}(\mathcal{A}_{\mathbf{nf}})\mathbf{Im}(\mathcal{A}_{\mathbf{sf}})],$$

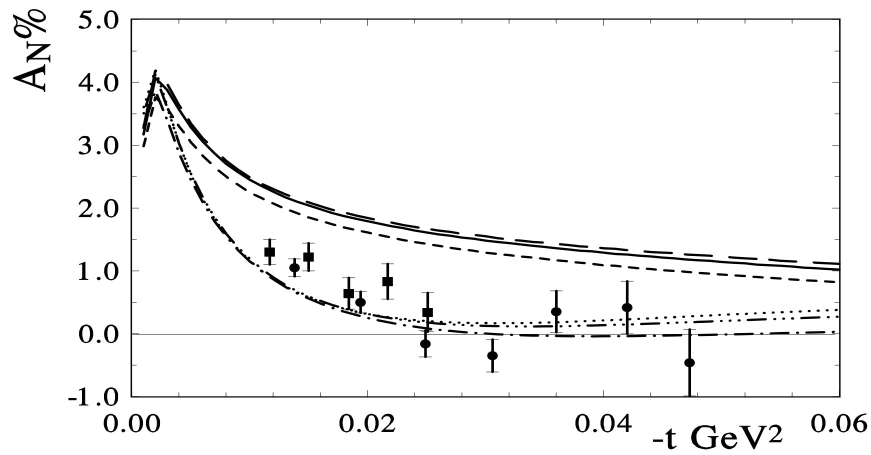
each term having electromagnetic and hadronic contributions.



a) The energy dependence of A_N (without hadron spin-flip) at $\mathbf{p_L} = 24, \ 100, \ 250$ GeV/c (dashed, solid, and long-dashed lines correspondingly) compared with the data at $\mathbf{p_L} = 22$ GeVc (cross) and $\mathbf{p_L} = 24$ GeV/c (boxes) .

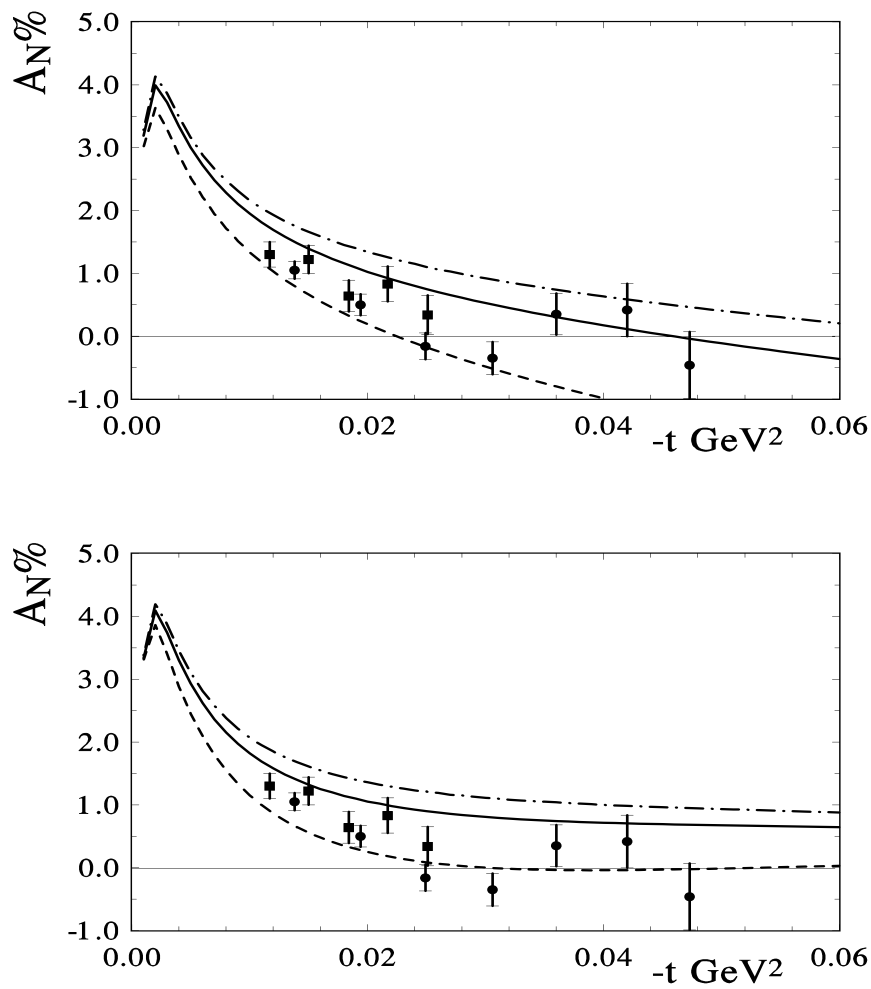
b) The dependence of $\mathbf{A_N}$ (without hadron spin-flip) over B^+ (solid line - B^+ is normalized to the data of A. Shiz.; dashed-points line - B^+ is normalized to the data of SELEX) compared with the data at $\mathbf{p_L} = 100$ GeVc (boxes and circles)





a) A_N with hadron-spin-flip amplitude in case I ($B^- = B^+$) for $p_L = 24,\ 100,\ 250$ GeV/c. (dash-dot, dashed-dots, and dots correspondingly).

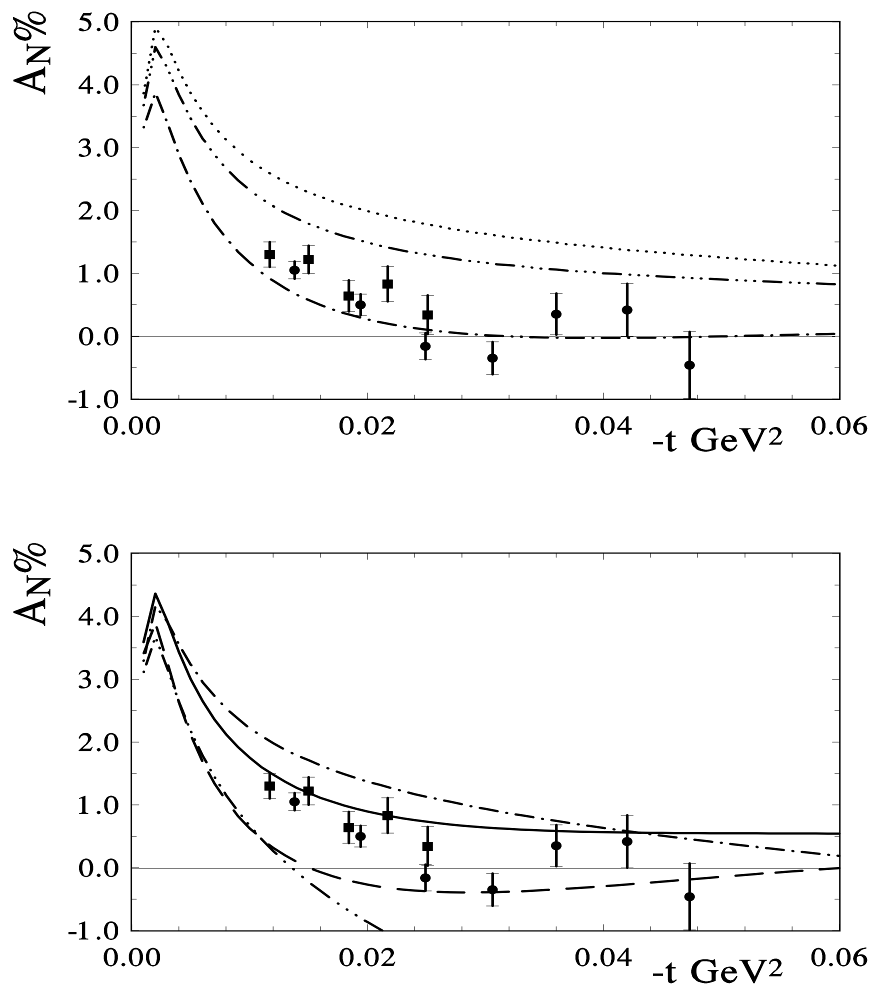
b) A_N with hadron-spin-flip amplitude in case II ($B^- = 2B^+$) for $p_L = 24,\ 100,\ 250$ GeV/c. (dash-dot, dashed-dots, and dots correspondingly).



A_N **with hadron spin-flip amplitude which has an additional energy factor** $\sqrt{s_1}/\sqrt{s}$

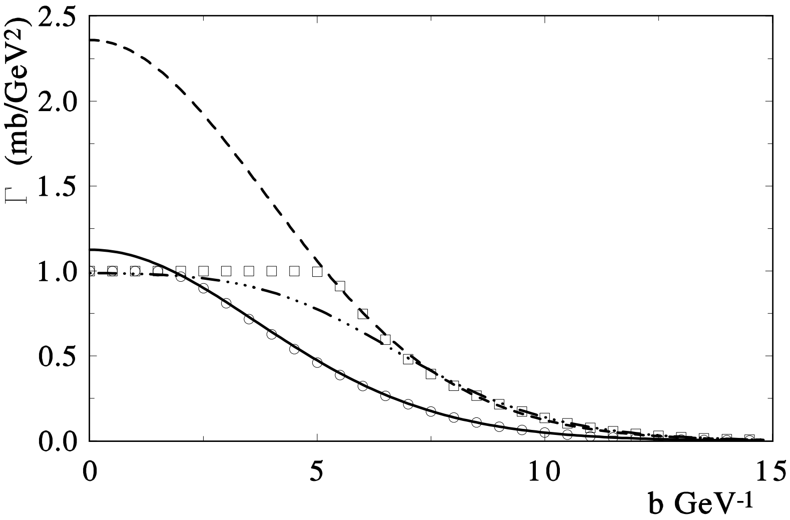
a) in case I ($\mathrm{B}^- = \mathrm{B}^+$) **for** $\mathrm{P_L} = 24, 100, 250$ **GeV**/c. **(dash-dot, dashed-dots, and dots correspondingly).**

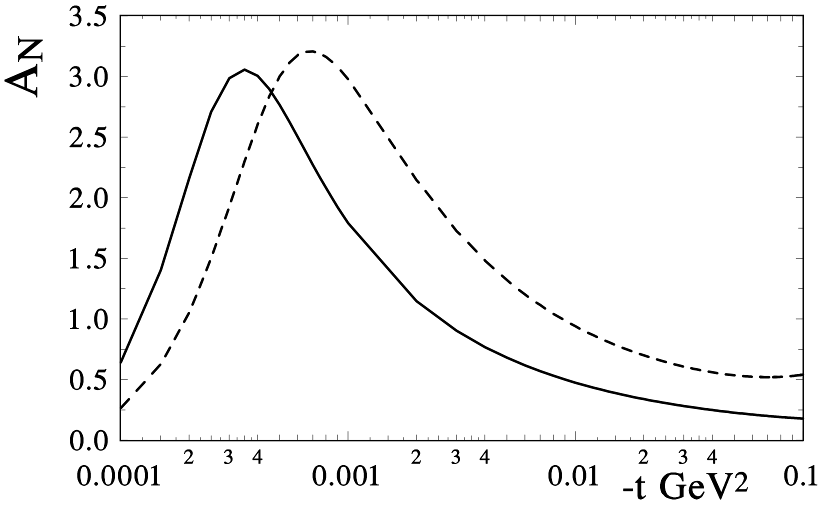
b) in case II ($B^- = 2B^+$) **for** $\mathrm{P_L} = 24,\ 100,\ 250$ **GeV**/c. **(dash-dot, dashed-dots, and dots correspondingly).**



a) A_N in case II $\mathrm{B}^{+-} = 2\mathrm{B}^{++}$ and with the real part of the spin-flip amplitude proportional to ρ at $\mathrm{p_L} = 24,\ 100,$ and 250 GeV/c. (dash-dot, dashed-dots, and dots correspondingly).

b) The effect on A_N of changing k_2 by ± 0.04 in cases (I) and (II) (dash-dot and dash-dots lines are for the case I; the solid and dash are for case II), for $\mathrm{p_L} =$ 100 GeV/c.


Γ (mb/GeV2)
2.5
2.0
1.5
1.0
0.5
0.0
0
5
10
15
b GeV-1



A_N
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0001
2
3
4
0.001
2
3
4
0.01
2
3
4
0.1
-t GeV2

S U M M A R Y

* The hadron-hadron interaction at large distances leads to a difference between the slopes of the hadron spin-flip amplitude and of the spin-non-flip amplitude.

* It is shown that the behavior of proton-nucleus elastic scattering at small momentum transfer at high energy can differ from the standard Glauber behavior.

* The new experimental data on A_N in $p^{12}C$ scattering at 24 and 100 GeV/c show a large slope for the residual spin-flip amplitude.

* It is very important to obtain data on A_N at larger energy and at $|t| \sim 0.06$ GeV^2 .

* Saturation effects, which are expected in proton-proton scattering at LHC energies, can be important in the case of proton-nucleus scattering already in the region of 100 GeV.

* The saturation effects lead to suppression of A_N at small t. This effect, and the impact of the saturation effects in the dip region, need in further investigation.