The role of Cahn and Sivers effects in Deep Inelastic Scattering

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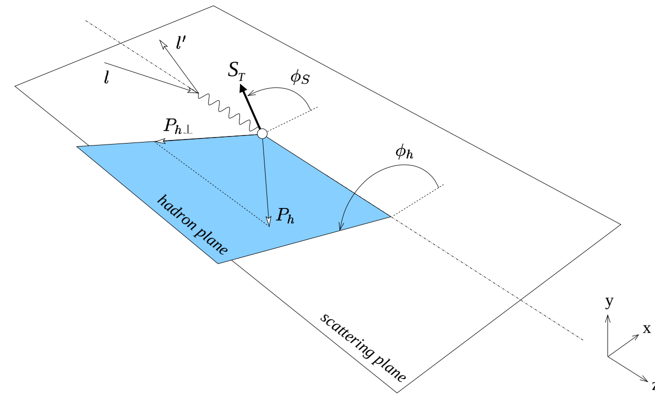
The plan of the talk

- Unpolarized Semi Inclusive Deep Inelastic Scattering (SIDIS)
  - Intrinsic $k_\perp$
  - Cahn effect
  - P_T^2 dependences
- Single Spin Asymmetries (SSA)
  - Sivers effect
  - The model for Sivers functions
  - Description of **HERMES (DESY)** data
- Conclusions

Unpolarized SIDIS

Cross section of **SIDIS**

$$\mathrm{d}\sigma^{lp\to lhX} = \sum_q f_q(x,Q^2) \otimes \mathrm{d}\sigma^{lq\to lq} \otimes D_q^h(z,Q^2)\ ,$$

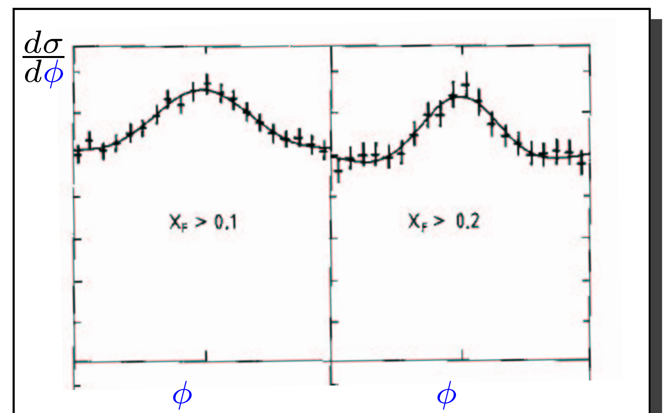


where f_q is the parton q density function, D_q^h is the fragmentation function of parton q into a hadron h.

In collinear parton model we have

$$\sigma^{lq\to lq} \propto \hat{s}^2 + \hat{u}^2 \propto 1 + (1-y)^2$$

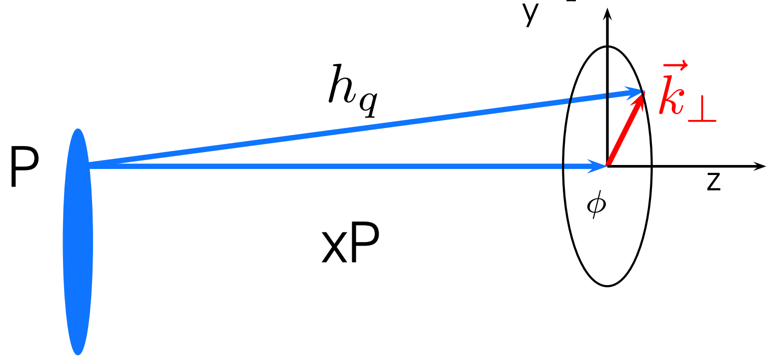
thus no dependence on azimuthal angle ϕ_h at first order of PT.
The experimental data reveal that $\mathrm{d}\sigma^{lp\to lh^\pm X}/\mathrm{d}\phi_h \propto \mathrm{A} + \mathrm{B}\cdot\cos(\phi_\mathrm{h}) + \mathrm{D}\cdot\cos(2\phi_\mathrm{h})$



M. Arneodo et al (EMC): Measurement of hadron azimuthal distributions, Z. Phys. C 34 (1987) 277

Intrinsic $k_\perp$, Cahn effect

Robert Cahn[1] introduced parton intrinsic transverse momentum $k_\perp$, parton momentum $h_q = xP + k_\perp$, where $k_\perp = (0, k_\perp cos(\phi), k_\perp sin(\phi), 0)$



$$\hat{s} = sx\left[1 - \frac{2k_\perp}{Q}\sqrt{1-y}\cdot\cos(\phi)\right] + \mathcal{O}\left(\frac{k_\perp^2}{Q}\right)$$

$$\hat{u} = sx(1-y)\left[1 - \frac{2k_\perp}{Q\sqrt{1-y}}\cdot\cos(\phi)\right] + \mathcal{O}\left(\frac{k_\perp^2}{Q}\right)$$

Hence (assuming collinear fragmentation)

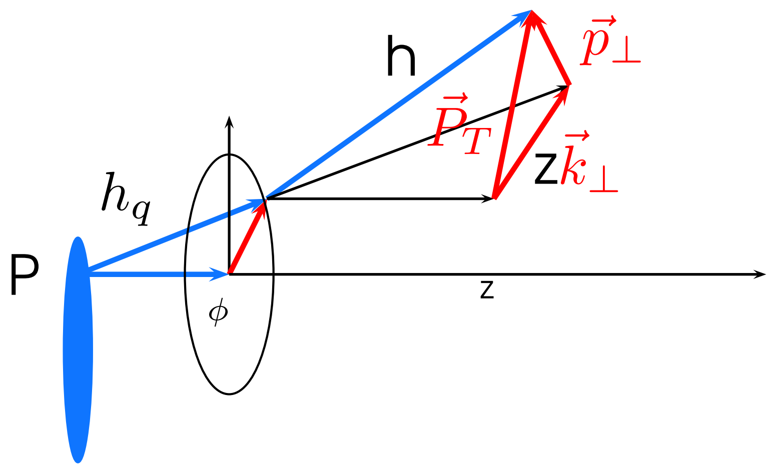
$$\frac{d\sigma_{ep\to ehX}}{d\phi_h} \propto \int dk_\perp^2 (\hat{s}^2 + \hat{u}^2) f_q(x, k_\perp^2) D_h^q(z) \propto \mathrm{A} + \mathrm{B}\cdot\cos(\phi_\mathrm{h}) + \mathrm{D}\cdot\cos(2\phi_\mathrm{h})$$

and these modulations of the cross section with azimuthal angle are called **Cahn effect**.

[1] R. Cahn, Phys. Lett. B 78 (1978) 269; Phys. Rev. D 40 (1989) 3107

Intrinsic $k_\perp$, Cahn effect

The situation is more complicated as the produced hadron may also have intrinsic transverse momentum with respect to the fragmenting parton.



Hadron transverse momentum $\vec{P}_T = \vec{p}_\perp + z\vec{k}_\perp$, and $\vec{p}_\perp$ is an analog of $\vec{k}_\perp$ but enters in D_h^q.

$$f_q(x, \vec{k}_\perp) \equiv f_q(x, k_\perp^2) \; ,$$
$$D_h^q(x, \vec{p}_\perp) \equiv D_h^q(x, p_\perp^2) \; ,$$

we obtain

$$\frac{d\sigma_{ep\to ehX}}{d\phi_h} \propto \int \mathrm{d}^2 k_\perp \{[1 + (1-y)^2] f_q(x, k_\perp^2) D_h^q(z, (\vec{P}_T - z\vec{k}_\perp)^2) -$$
$$-4\sqrt{1-y}(2-y)\frac{k_\perp cos(\phi)}{Q} f_q(x, k_\perp^2) D_h^q(z, (\vec{P}_T - z\vec{k}_\perp)^2)\} + \mathcal{O}\left(\frac{k_\perp^2}{Q^2}\right)$$

Intrinsic $k_\perp$, Cahn effect

Let us assume that $k_\perp$ and $p_\perp$ distributions have the following form

$$f_q(x, k_\perp^2) = f_q(x) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-\frac{k_\perp^2}{\langle k_\perp^2 \rangle}} \,,$$

$$D_h^q(x, p_\perp^2) = D_h^q(x) \frac{1}{\pi \langle p_\perp^2 \rangle} e^{-\frac{p_\perp^2}{\langle p_\perp^2 \rangle}} \,,$$

then we can integrate the previous formula and obtain

$$\frac{\mathrm{d}^5 \sigma^{ep \to ehX}}{\mathrm{d}x \mathrm{d}y \mathrm{d}z P_T \mathrm{d}P_T \mathrm{d}\phi_h} \propto \{[1 + (1-y)^2] - 4 \frac{\sqrt{1-y}(2-y)\langle k_\perp^2 \rangle z P_T}{(\langle p_\perp^2 \rangle + z^2 \langle k_\perp^2 \rangle) Q} \cos(\phi_\mathrm{h})\} \cdot$$

$$\cdot f_q(x) D_h^q(z) \frac{1}{\pi \langle P_T^2 \rangle} e^{-\frac{P_T^2}{\langle P_T^2 \rangle}} \,,$$

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z^2 \langle k_\perp^2 \rangle$$

$\langle p_\perp^2 \rangle$&$\langle k_\perp^2 \rangle$ are essential ingredients for SSA in SIDIS.
One must describe the data on unpolarized SIDIS before describing SSA.

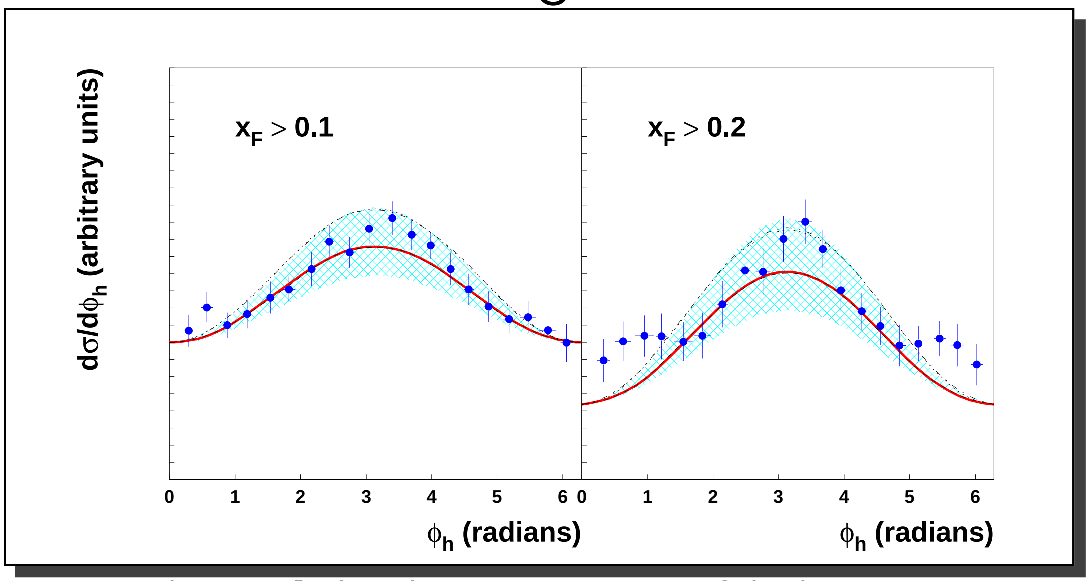
Cahn effect

We choose the values for parameters $\langle k_\perp^2 \rangle$ and $\langle p_\perp^2 \rangle$:

$\langle k_\perp^2 \rangle = 0.25$ GeV^2

$\langle p_\perp^2 \rangle = 0.2$ GeV^2

and obtain the following description of angular dependence: ϕ_h dependence was measured by **EMC** at **CERN** in μp and μd scattering at incident beam energies between 100 and 280 GeV.



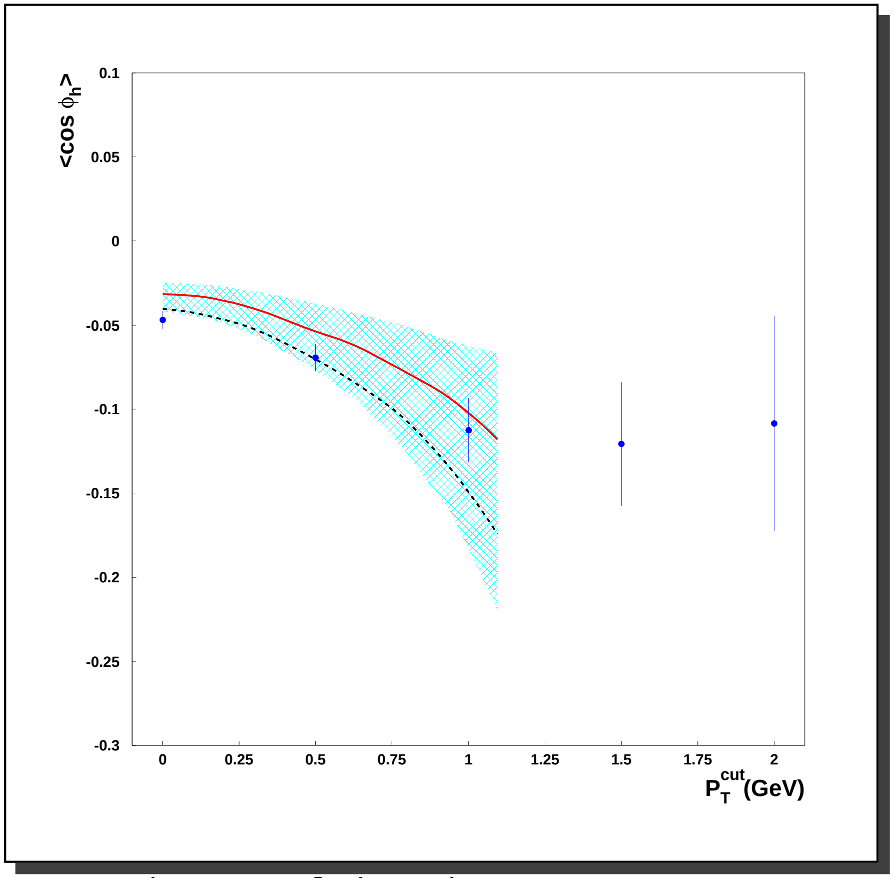
M. Arneodo et al (EMC): Measurement of hadron azimuthal distributions, Z. Phys. C 34 (1987) 277

The red line corresponds to complete kinematics, the dashed line up to $\mathcal{O}\left(\frac{k_\perp}{Q}\right)$ terms, the shadowed region corresponds to varying $\langle k_\perp^2 \rangle$ and $\langle p_\perp^2 \rangle$ by 20%

Cahn effect

Another feature measured in experiment is

$$\langle \cos(\phi_h) \rangle = \frac{\int \sigma \cos(\phi_h) d\phi_h}{\int \sigma d\phi_h} = \frac{B}{2A}$$



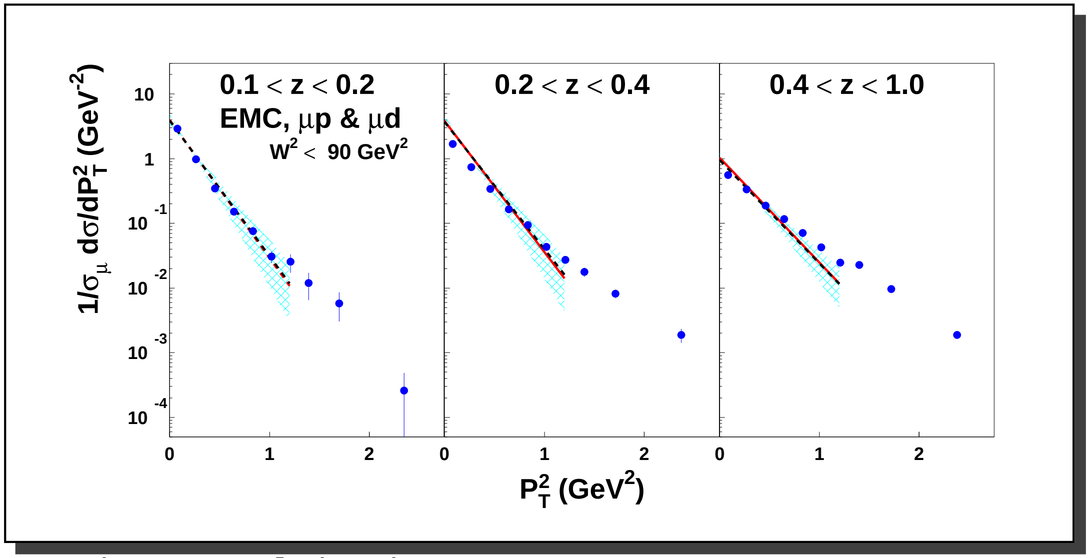
M. R. Adams et al (E665)

Phys. Rev D 48 (1993) 5057

The data are from **E665** at Fermilab, $E_{lab} = 490$ GeV. At high P_T^{cut} (σ is integrated on P_T from P_T^{cut} to P_T^{max}) the contribution from nonperturbative intrinsic momentum is small and $\gamma^* q \to qg$, $\gamma^* g \to q\bar{q}$ and other perturbative QCD effects dominate.
The red line corresponds to complete kinematics, the dashed line up to $\mathcal{O}\left(\frac{k_\perp}{Q}\right)$ terms, the shadowed region corresponds to varying $\langle k_\perp^2 \rangle$ and $\langle p_\perp^2 \rangle$ by 20%

P_T^2 dependence

P_T^2 dependence was measured by **EMC** at **CERN** in μp and μd scattering at incident beam energies between 100 and 280 GeV.



J. Ashman et al (EMC)

Z. Phys. C 52 (1991) 361-387

At first order

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z^2 \langle k_\perp^2 \rangle$$

The red line corresponds to complete kinematics, the dashed line up to $\mathcal{O}\left(\frac{k_\perp}{Q}\right)$ terms, the shadowed region corresponds to varying $\langle k_\perp^2 \rangle$ and $\langle p_\perp^2 \rangle$ by 20%

We conclude that using the unpolarised data one can fix the values for parameters $\langle k_\perp^2 \rangle$ and $\langle p_\perp^2 \rangle$:

$\langle k_\perp^2 \rangle = 0.25$ GeV^2

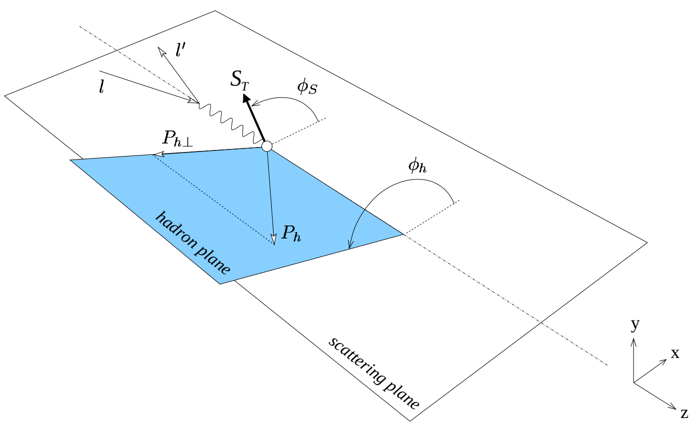
$\langle p_\perp^2 \rangle = 0.2$ GeV^2

Polarized SIDIS and Sivers effect

Cross section of polarized **SIDIS**

$$d\sigma^{lp^\uparrow \to lhX} = \sum_q f_{q/p^\uparrow}(x, Q^2) \otimes d\sigma^{lq^\uparrow \to lq^\uparrow} \otimes D^h_{q^\uparrow}(z, Q^2)$$

where $f_{q/p^\uparrow}$ is the parton q density function, $D^h_{q^\uparrow}$ is the fragmentation function of parton q into a hadron h. The structure of cross sections becomes more complicated due to presence of new fragmentation and density functions.



An asymmetry is defined as $A = \frac{d\sigma^{\Uparrow} - d\sigma^{\Downarrow}}{d\sigma^{\Uparrow} + d\sigma^{\Downarrow}}$
Let us consider a particular case of azimuthal modulations in parton density distribution, so called **Sivers effect**.

see, for example, A. Kotzinian Nucl. Phys. B 441 (1995) 234-356

SIVERS EFFECT

Unpolarized quark distributions inside a transversely polarized proton may be written as

$$f_{q/p^\uparrow}(x,\boldsymbol{k}_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\mathbf{S_T}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)\;,$$

where $\Delta^N f_{q/p^\uparrow}(x,k_\perp)$ is so called Sivers function which must comply with the following positivity bound

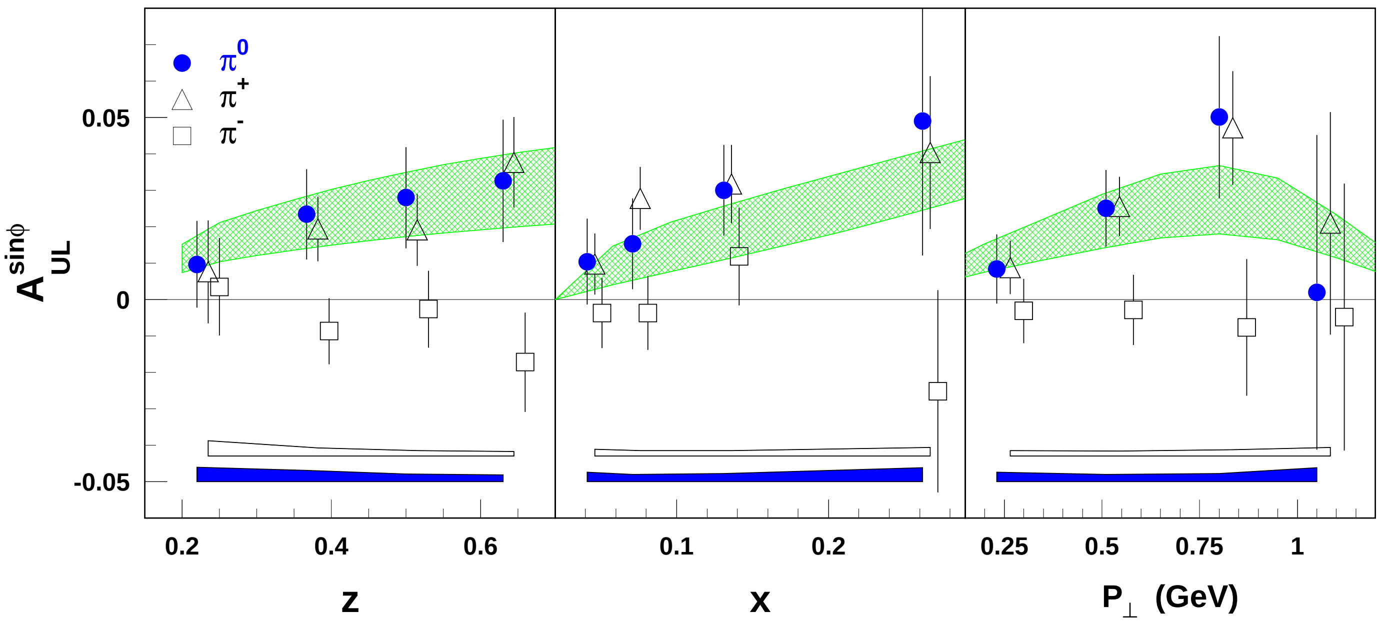
$$\frac{\Delta^N f_{q/p^\uparrow}(x,k_\perp)}{2 f_{q/p}(x,k_\perp)} \leq 1$$

Arising SSA has the following form

$$A = \frac{\sum_q \int d^2\boldsymbol{k}_\perp \Delta^N f_{q/p^\uparrow}(x,k_\perp)\mathbf{S_T}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)\frac{d\sigma^{lq\to lq}}{dQ^2} J\frac{z}{z_h} D_q^h(z,\mathbf{p}_\perp)}{2\sum_q \int d^2\boldsymbol{k}_\perp f_q(x,k_\perp)\frac{d\sigma^{lq\to lq}}{dQ^2} J\frac{z}{z_h} D_q^h(z,\mathbf{p}_\perp)}$$

Experimental situation

Single spin asymmetry $A_{UL}^{sin(\phi_h)} = \frac{\int (d^6\sigma^{\uparrow} - d^6\sigma^{\downarrow}) sin(\phi_h) d\phi_h}{\int 1/2(d^6\sigma^{\uparrow} + d^6\sigma^{\downarrow}) d\phi_h}$



A. Airapetian et al, Phys.Rev. D64 (2001) 097101

A. Airapetian et al, Phys. Rev. Lett. 84 (2000) 4047-4051

Both Sivers and Collins effects contribute to $A_{UL}^{sin(\phi_h)}$.
Let us check if it is possible to describe the data using the Sivers effect only.

The model for the Sivers function

Let us use the following form for the Sivers functions:

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = N_q(x) h(k_\perp) f_{q/p}(x, k_\perp) \; ,$$

Where $f_{q/p}(x)$ is parton q distribution function,

$$N_q(x) = N_q x^{a_q} (1-x)^{b_q} \frac{(a_q + b_q)^{(a_q+b_q)}}{a_q^{a_q} b_q^{b_q}} \; ,$$

$$h(k_\perp) = \frac{2 k_\perp M}{k_\perp{}^2 + M^2} \; ,$$

where

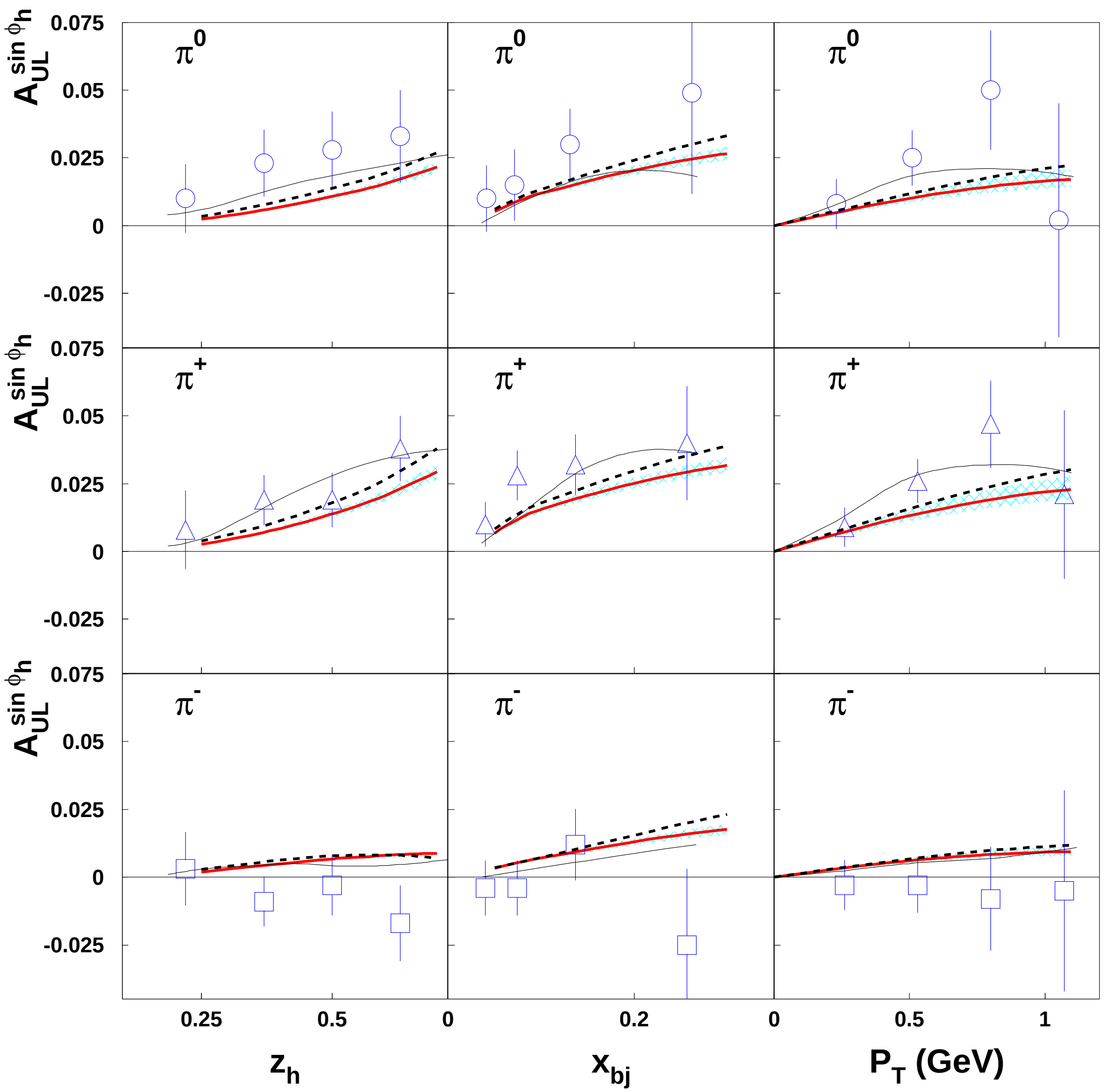
$M^2 = \langle k_\perp^2 \rangle = 0.25$ GeV^2 ,

$N_u = -1, a_u = 0.1$, $b_u = 0.3$

$N_d = \quad 1, a_d = 0.1$, $b_d = 0.3$

For all other quarks Sivers function is zero.

Description of $A_{UL}^{sin(\phi_h)}$



PDF: MRST LO 98

Eur. Phys. J. C4 (1998) 463

Fragmentation functions: Kretzer

Phys. Rev. D62 (2000) 054001

$ep \to e\pi X$

$$Q^2 > 1\,\mathrm{GeV}^2\,, \quad W^2 > 10\,\mathrm{GeV}^2\,,$$

$$0.023 < x_{Bj} < 0.4\,, \quad 0.2 < z_h < 0.7$$

The solid line correspond to calculation with the use of event generator. The red line corresponds to complete kinematics, the dashed line up to $\mathcal{O}\left(\frac{k_\perp}{Q}\right)$ terms

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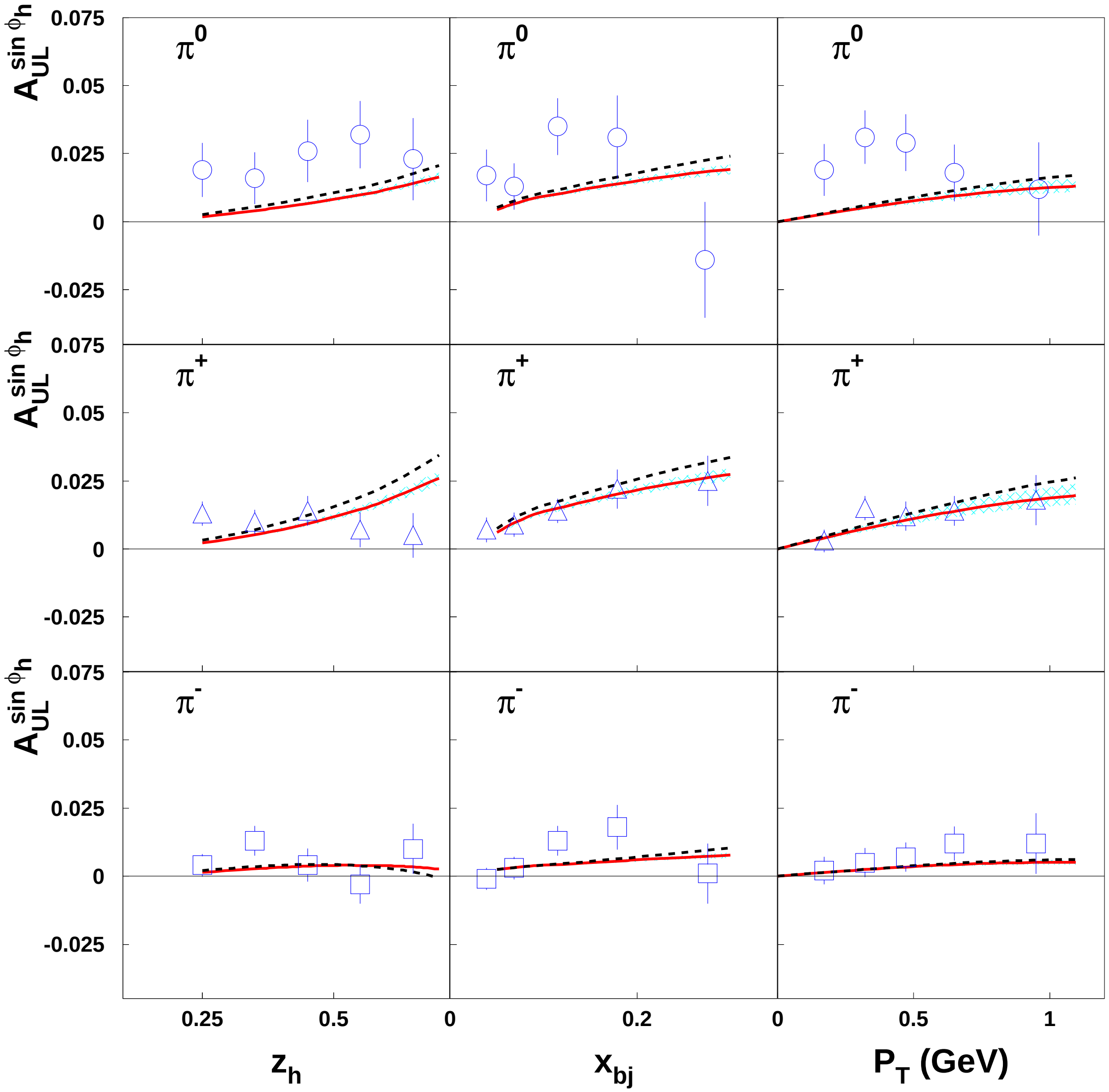


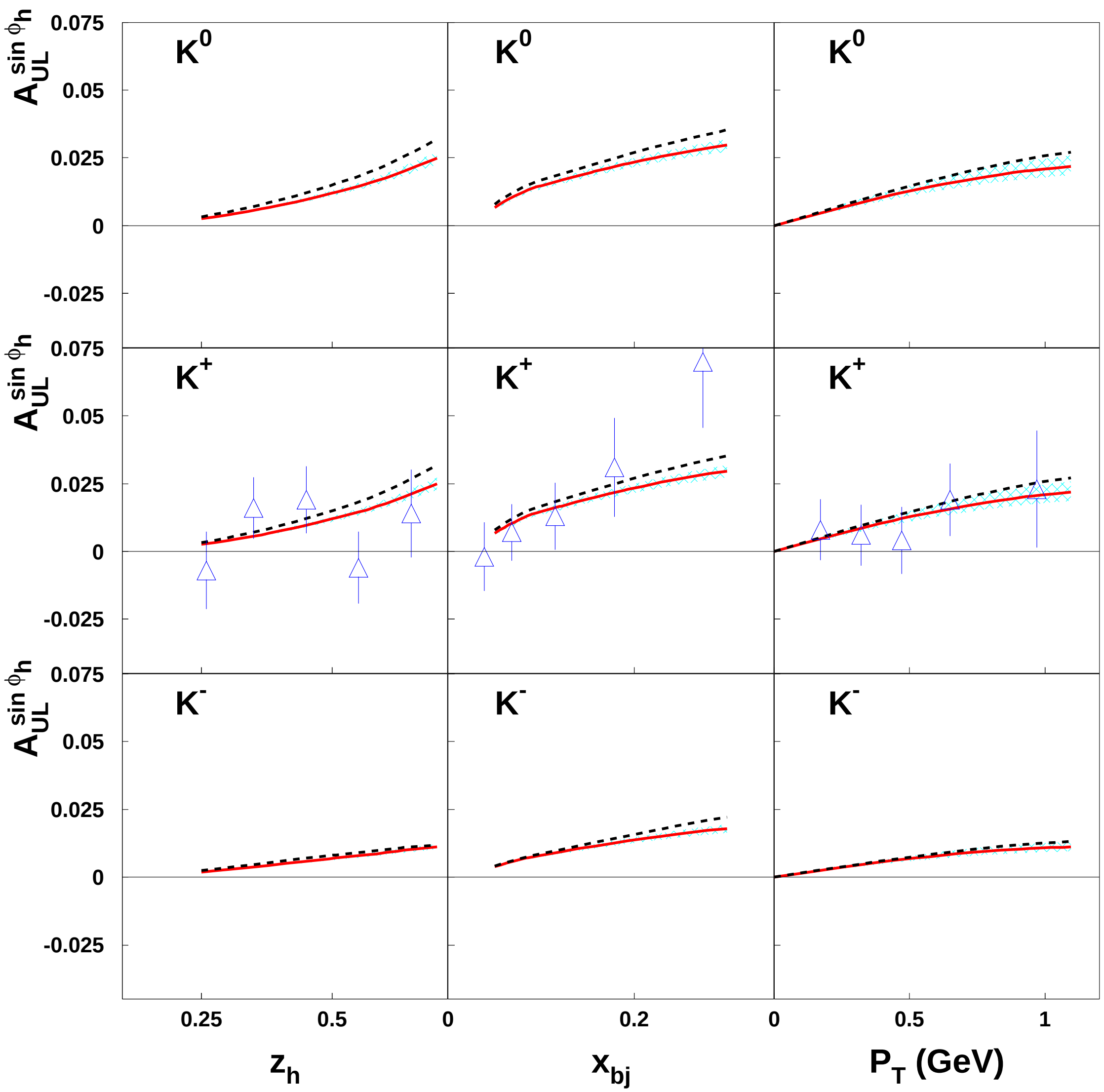
$eD \to e\pi X$

All parameters are fixed

Hermes data (open symbols) on $A_{UL}^{sin(\phi)}$. The red line corresponds to complete kinematics, the dashed line up to $\mathcal{O}\left(\frac{k_\perp}{Q}\right)$ terms

A. Airapetian et al, Phys. Lett. B562 (2003) 182

Description of $A_{UL}^{sin(\phi_h)}$



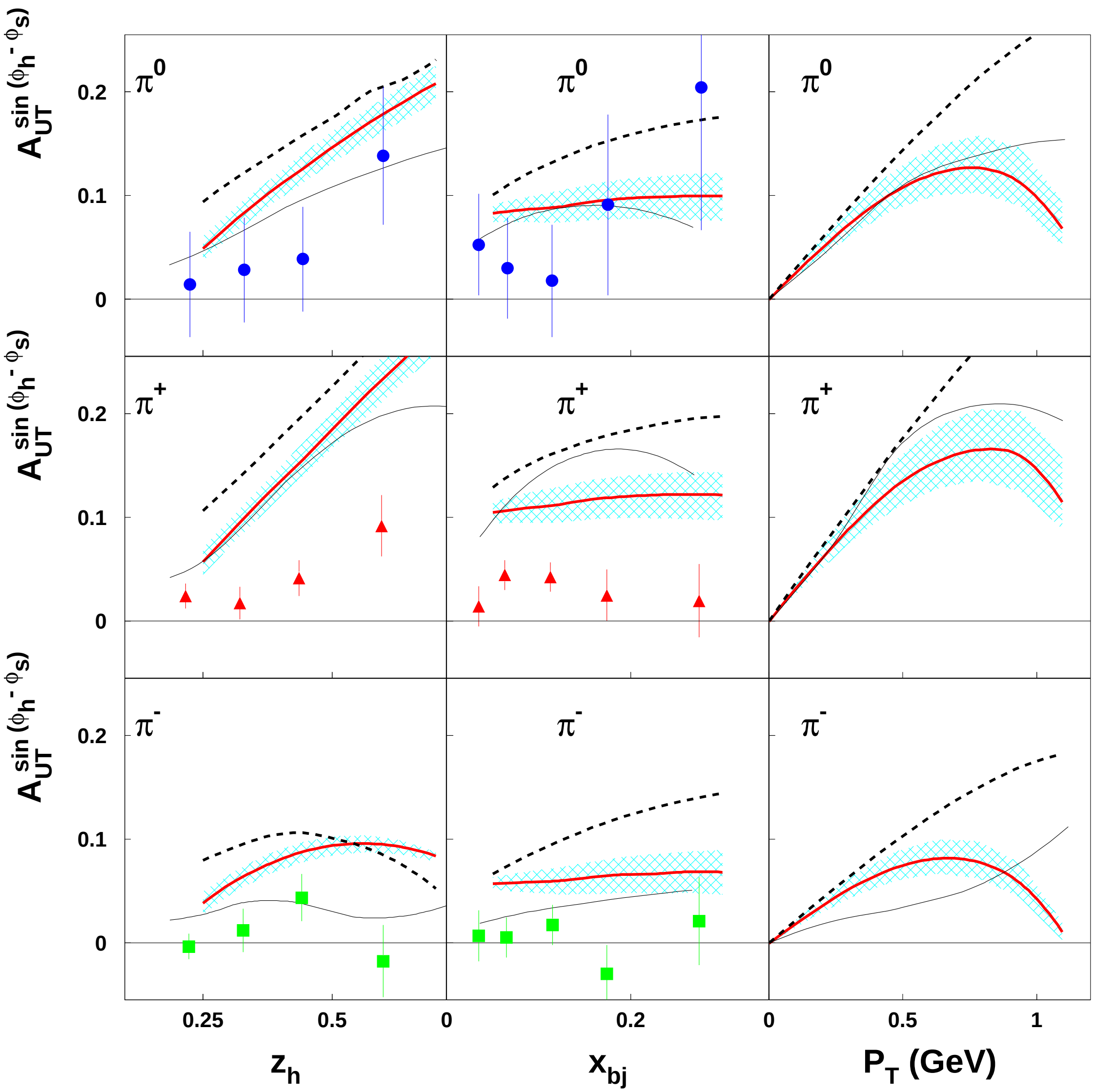
$eD \to eKX$

<u>All parameters are fixed</u>

Hermes data (open symbols) on $A_{UL}^{sin(\phi)}$. The red line corresponds to complete kinematics, the dashed line up to $\mathcal{O}\left(\frac{k_\perp}{Q}\right)$ terms

A. Airapetian et al, Phys. Lett. B562 (2003) 182

Description of $A_{UT}^{sin(\phi_h - \phi_S)}$



$ep \to e\pi X$

The red line corresponds to the complete kinematics, the dashed line up to $\mathcal{O}\left(\frac{k_\perp}{Q}\right)$.

N.C.R. Makins, First Transverse Target Data from HERMES, Transversity Workshop, Athens, Greece, Oct 6 - 7, 2003

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''Single-spin asymmetries in semi-inclusive deep-inelastic scattering on a transversely polarised hydrogen target'' Hermes collaboration

hep-ph/0408013

CONCLUSIONS & PLANS

- It is shown that the model with intrinsic $k_\perp$ and $\langle k_\perp^2 \rangle = 0.25$ GeV^2, $\langle p_\perp^2 \rangle = 0.20$ GeV^2 is capable of reproducing the unpolarized SIDIS data.

- It is shown that SSA $A_{UL}^{sin(\phi_h)}$ may be described using Sivers effect only.

- Sivers functions derived from the analysis of $A_{UL}^{sin(\phi_h)}$ overestimate SSA on a trasversely polarized target $A_{UT}^{sin(\phi_h-\phi_S)}$ $\rightarrow$ nonzero Collins effect?

- Sivers functions derived from the analysis of $A_{UL}^{sin(\phi_h)}$ are different from those obtained in the analysis of SSA in $p^\uparrow p \rightarrow \pi X$.
  - An analysis including Sivers and Collins effect is needed.
  - QCD corrections must be taken into account in order to describe the data at high P_T.
  - An analysis of SIDIS and DY must be performed.

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Stay tuned! More results to come.