PARITY OF THE Θ^+ PENTAQUARK AND SPIN OBSERVABLES FOR THE REACTION $NN \to Y\Theta^+$

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PLAN:

- Θ^+ -?
- **Phenomenology $NN \to Y\Theta^+$ at the threshold**
- **Nonstandard method, $C_{y,y}$**
- **General method, arbitrary spins**
- **σ-representation $\vec{\frac{1}{2}} + \vec{\frac{1}{2}} \to \vec{\frac{1}{2}} + \vec{\frac{1}{2}}$**
- **Conclusion**

Θ^+ is explicitly exotic ($uudd\bar{s}$)

$S=+1 \quad B=1$

$\Sigma^{--} \quad S=-2 \quad Q=-2$

($\Sigma^{+} \quad S=-2 \quad Q=+1$)

$\overline{10} + 8$

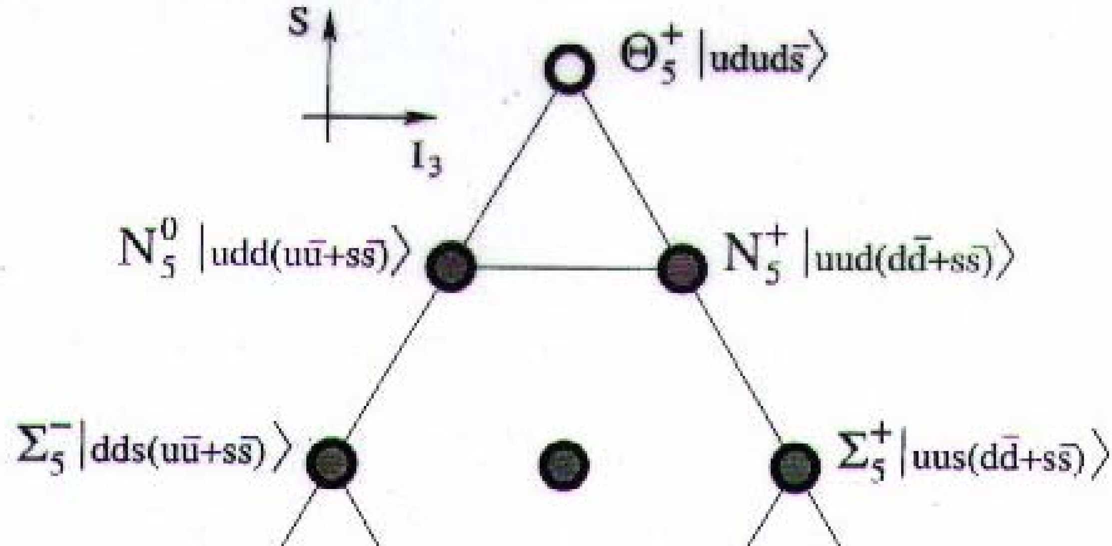


Figure 1: Quark structure of the antidecuplet (annuli) and octet (filled circles) that are generally assumed for the lowest-mass pentaquarks. Strangeness increases in the vertical direction and the third component of isospin in the horizontal.

Does Θ^+ exist?

Positive: – LEPS, DIANA, CLAS(d), SHAPIR, ITEP, CLAS(p), HERMES SVD COSY-TOF, JINR-BC, ZEUS, ...

* Θ^+ **S=+1 , B=1** nK^+, pK^0 $M = 1520-1540 MeV/c^2$, $\Gamma < 25$, 9, **or** $< 1 MeV$ (Θ^{++} **not observed** $\Longrightarrow I_\Theta = 0$)

* **S=** -2 $\approx 1862 MeV/c^2$, $\Gamma \leq 18 MeV/c^2$

$\Xi^{--}(sdsd\bar{u}), \Xi^0,$ $I = \frac{3}{2}$ **NA49**

* **uudd**$\bar{c}$ $3099\, MeV/c^2$ **H1 hep-exp/0403017** Θ_c^0

N^0 **or** Ξ^0 $I = \frac{1}{2}$ $1734 MeV/c^2$ ΛK_s^0

* **STAR RHIC, hep-ex/0406032**

Null results for Θ^+, $\Xi^{--}_{3/2}$, Θ_c: **BES, HERA-B, OPAL, PHENIX, DELPHI, ALEPH, HYPER-CP, E690, CDF, BABAR**

...

THEORY

Chiral soliton model Diakonov, Petrov, Polaykov

(ud)-(ud$\bar{s}$) Karliner-Lipkin

$(ud)^2$-$\bar{s}$ Jaffe-Wilczek

QCD sum rules

Lattice QCD

S.-L. Zhu (nucl-th/0410013) 04.10.2004

„... NONE of theoretical models predicts the existance of $\Theta(1530)$ reliably"

J.M. Richard, Plenary talk on 13 october SPIN 2004

P-parity of the Θ^+ (models)

Chiral soliton $\frac{1}{2}^+$

Non-correlated quarks
$(1S)^5$ $\pi = -1$

Chiral qq-interaction P-wave
$\pi = +1$ $\frac{1}{2}, \frac{3}{2}$

Color-magnetic qq-forces
$(ud)^2 \bar{s}$ $\frac{1}{2}^+$

Color-electric forces
$(ud)(ud\bar{s})$ $\frac{1}{2}^+$

Lattice QCD $\pi = -1$
-"-"- $(ud)^2 \bar{s}$ $\pi = +1$

T. W. Chiu, T. H. Hsieh
hep-ph/0403020

$$J^{\pi} = \frac{1}{2}^+ : \quad m_N[(ud)^2 \bar{d}] = 1460\ (51)$$
$$m_\Theta[(ud)^2 \bar{s}] = 1539\ (95)$$
$$m_\Xi[(ds)^2 \bar{u}] = 1826\ (87)$$

P_Θ -Parity determination in $NN \to Y\Theta^+$ (model-independent)

A.W. Thomas, K.Hicks, A.Hosaka,

Prog.Theor.Phys. 111 (2004) 291

$\vec{p}\,\vec{p} \to \Sigma^+\Theta^+$ near threshold

C.Hanhart et al. Phys.Lett. 590 (2004) 39;

$C_{x,x}$, $j_\Theta = \frac{1}{2}$

Yu.N.U. , Phys.Lett. B595(2004) 277; hep-ph/0402216.

$\vec{p}n \to \vec{\Lambda}^0\Theta^+$ arbitrary spin j_Θ $\vec{\Lambda}^0 \to p + \pi^-$

$\vec{p}p \to \vec{\Sigma}^+\Theta^+$, K_y^y

M.Rekalo, E.Tomasi-Gustaffson, Phys.Lett. B591 (2004) 225; hep-ph/0402227.

C.Hanhart, J.Haidenbauer, K.Nakayama, U.-G.Meissner, hep-ph/0407107 $K + K^*$ - exch.

The reaction $1 + 2 \to 3 + 4$ at the threshold

$$T^{\mu_3\,\mu_4}_{\mu_1\,\mu_2} = \sum_{\substack{J\,M \\ S\,M_S\,L\,m}} (j_1\mu_1\,j_2\mu_2|S M_S) \times$$

$$(j_3\mu_3\,j_4\,\mu_4|J\,M)\,(S\,M_S\,L\,m|J\,M)\,Y_{Lm}(\hat{\mathbf{k}})a^{LS}_J$$

$\mathbf{J} = \mathbf{j_3} + \mathbf{j_4},\ \mathbf{j_3} + \mathbf{j_4} - \mathbf{1},\ \ldots,\ |\mathbf{j_3} - \mathbf{j_4}|.$

$\vec{S} = \vec{j}_1 + \vec{j}_2$

$(-\mathbf{1})^{\mathbf{L}} = \boldsymbol{\pi}$, where $\boldsymbol{\pi} = \boldsymbol{\pi}_1\,\boldsymbol{\pi}_2\,\boldsymbol{\pi}_3\,\boldsymbol{\pi}_4$

$\vec{J} = \vec{S} + \vec{L}$

Pauli: $(-\mathbf{1})^{\mathbf{L+S+T}} = -\mathbf{1}$

$(-\mathbf{1})^{\mathbf{S}} = \pi(-\mathbf{1})^{\mathbf{T+1}}$

T=1 $(-1)^S = \pi$ **S=0 if** $\pi = +1$

* **S = 1 if** $\pi = -1$

T=0 $(-1)^S = -\pi$ **S=1 if** $\pi = +1$

* **S = 0 if** $\pi = -1$

At given T, $S \Rightarrow \pi$ independently of j_3, j_4

The NUMBER of AMPLITUDES a_J^{LS}

$j_1 = j_2 = j_3 = \frac{1}{2}$ **and** $j_4 = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots.$

$J_p = j_4 + \frac{1}{2}$, $J_m = j_4 - \frac{1}{2}$

S = 0

$a_J^{LS} = a_J^{J0}$, **where** $(-1)^L = \pi$.

$\boxed{\mathbf{S = 1}}$

For $j_4 \geq \frac{3}{2}$ **– three amplitudes** $a_J^{L1} \equiv a_J^L$**:**

- $a_{J_p}^{J_p}$, $a_{J_m}^{J_m+1}$ $a_{J_m}^{J_m-1}$, **if** $(-1)^{J_p} = \pi$,
- $a_{J_m}^{J_m}$, $a_{J_p}^{J_p+1}$ $a_{J_p}^{J_p-1}$, **if** $(-1)^{J_p} = -\pi$.

For $j_4 = \frac{1}{2}$ **– two amplitudes:**

$J = 1$, $L = 0$, $L = 2$, **if** $\pi = +1$,

$J = 0$, $J = 1$, $L = 1$, **if** $\pi = -1$

NONSTANDARD METHOD

The polarized cross section

$$d\sigma(\mathbf{p}_1, \mathbf{p}_2) = \Phi \sum_{\mu_3 \mu_4} |T^{\mu_3 \mu_4}_{\mu_1 \mu_2}|^2 =$$

$$= \frac{1}{4\pi} \sum_M \underbrace{(\frac{1}{2}\mu_1 \frac{1}{2}\mu_2 | SM)^2} \sum_{J\,M\,L\,L'} \sqrt{(2L+1)(2L'+1)} \times$$
$$\times (S\,M\,L\,0|J\,M)(S\,M\,L'\,0|J\,M) a_J^{LS} (a_J^{L'S})^*,$$

Using the relations

$$(\frac{1}{2}\mu_1 \frac{1}{2}\mu_2 | 00) = \chi^+_{\mu_1} \frac{i\sigma_y}{\sqrt{2}} \chi^{(T)+}_{\mu_2}$$

$$(\frac{1}{2}\mu_1 \frac{1}{2}\mu_2 | 1\lambda) = \chi^+_{\mu_1} \sigma_\lambda \frac{i\sigma_y}{\sqrt{2}} \chi^{(T)+}_{\mu_2}$$

$\sigma_i\,(i = y, \lambda)$ is the Pauli matrix

$$(\frac{1}{2}\mu_1 \frac{1}{2}\mu_2 | 00)^2 = \frac{1}{4}(1 - \mathbf{p}_1 \cdot \mathbf{p}_2), \qquad (3)$$

$$(\frac{1}{2}\mu_1 \frac{1}{2}\mu_2 | 1M)^2 = \begin{cases} \frac{1}{4}(1 + \mathbf{p}_1 \cdot \mathbf{p}_2 - 2p_{1z}p_{2z}), \; M = 0, \\ \frac{1}{4}[1 \pm (p_{1z} + p_{2z}) + p_{1z}p_{2z}], \; M = \pm 1, \end{cases} \qquad (4)$$

The spin singlet initial state

$S = 0$

$$\boxed{d\sigma(\mathbf{p}_1, \mathbf{p}_2) = d\sigma_0(1 - \mathbf{p}_1 \cdot \mathbf{p}_2), \qquad (5)}$$

$$C_{x,x} = C_{y,y} = C_{z,z} = -1$$

.

The unpolarized cross section is given as

$$d\sigma_0 = \Phi \frac{1}{4} \sum_{\mu_1 \mu_2 \mu_3 \mu_4} |T^{\mu_3 \mu_4}_{\mu_1 \mu_2}|^2 = \frac{1}{16\pi} \Phi \sum_{J,L} (2J+1) |a^{L\,S}_J|^2. \tag{6}$$

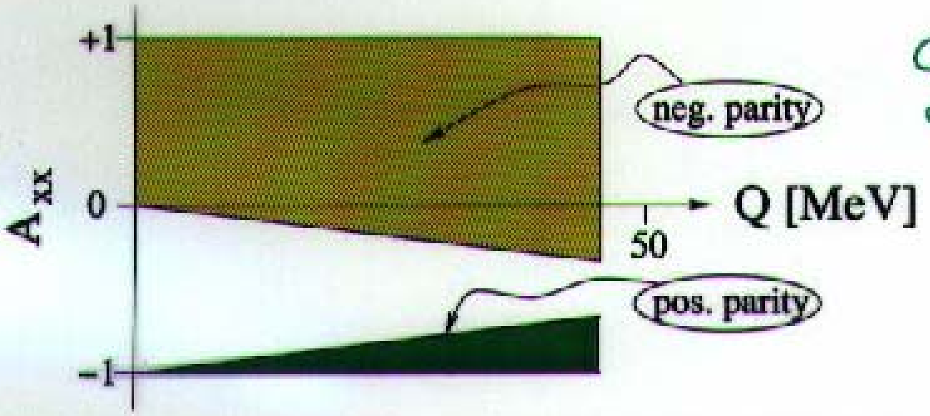
Spin-spin correlation for $\vec{\frac{1}{2}} + \vec{\frac{1}{2}} \to j_3 + j_4$

$S=1$

$$d\sigma(\mathbf{p}_1, \mathbf{p}_2) = d\sigma_0\,(1 + C_{x,x}\,p_{1x}\,p_{2x} + C_{y,y}\,p_{1y}p_{2y} + C_{z,z}\,p_{1z}\,p_{2z})$$

for arbitrary j_3 and j_4:

$$\boxed{\begin{aligned} C_{x,x} &= C_{y,y} = \frac{\sum_J |\sqrt{J}\,a_J^{J-1} - \sqrt{J+1}\,a_J^{J+1}|^2}{\sum_{J\,L}(2J+1)\,|a_J^L|^2}, \geqslant 0 \\ C_{z,z} &= 1 - 2\,C_{y,y} \end{aligned}}$$



C. Hanhart et al, PL B590 (2004)

$j_\Theta = \frac{1}{2}$

Fig. 2. Schematic presentation of the result for A_{xx} for the two possible parity states of the Θ^+. For either option realized the corresponding data should fall into the area indicated. In case of a negative parity the threshold value depends on the ratio of the strength of the two possible s–wave amplitudes.

given above should be applicable for $Q < 50$ MeV. The expected signal for A_{xx} is sketched in Fig. 2.

Implicitly we assumed that there is no strong $\Theta^+\Sigma^+$ final state interaction that would introduce an additional large scale into the system. This is justified, because most of those meson exchanges that potentially could lead to a strong final–state interaction are either absent or should be weak: (i) a single pion exchange between Θ^+ and Σ^+ is not possible due to the isoscalar nature of the pentaquark, (ii) a strong coupling of the Θ^+ to NK is excluded due to its small width and (iii) there can also be no strong coupling of the Θ^+ to the iso–scalar two pion exchange, known to be responsible for the medium range attraction of the NN interaction, since then the Θ^+ should not be seen equally narrow in nuclear reactions [2] and in elementary production reactions on a single nucleon [4,5].

4. We also want to briefly comment on the possible influence of the background on the signal. In principle there is the admittedly rather unlikely possibility that the pentaquark signal observed does—at least to some extend—stem from an interference of the Θ^+ production amplitude with the background. How would this change our analysis? To simplify this discussion we will assume that the observables are fully angular integrated. Thus, we do not have to worry about the interference amongst different partial waves. In addition, we will only discuss the observables in threshold kinematics.

The observable ${}^3\sigma_\Sigma$ defined in Eq. (2) is a projector on spin triplet initial states irrespective of the final states. Thus, even if the observed signal would be due to an interference of a positive parity Θ^+ amplitude with the background, A_{xx} would approach -1 as the energy approaches the Θ^+ production threshold.

The situation is a little more complicated for the negative parity Θ^+, for here two amplitudes contribute at threshold. In this case the threshold amplitude

General method

S.M. Bilenky, L.L. Lapidus, L.D. Puzikov and R.M. Ryndin, Nucl. Phys. **7** (1958) 646.

$$T_{\mu_1 \mu_2}^{\mu_3 \mu_4} = \chi_{j_3 \mu_3}^{+} \chi_{j_4 \mu_4}^{+} \hat{F} \chi_{j_1 \mu_1} \chi_{j_2 \mu_2}, \qquad (14)$$

$$\hat{F} = \sum_{\substack{m_1 m_2 \\ m_3 m_4}} T_{m_1 m_2}^{m_3 m_4} \chi_{j_1 m_1}^{+}(1) \chi_{j_2 m_2}^{+}(2) \chi_{j_3 m_3}(3) \chi_{j_4 m_4}(4), \qquad (15)$$

$$d\sigma_0 = \frac{\Phi}{(2j_1+1)(2j_2+1)} Sp F F^{+}. \qquad (16)$$

The spin-transfer coefficient

$\overrightarrow{\frac{1}{2}} + j_2 \to \overrightarrow{\frac{1}{2}} + j_4$

$$K_\lambda^\kappa = \frac{SpF\sigma_\lambda(1)\,F^+\,\sigma_\kappa(3)}{SpF\,F^+}, \qquad (17)$$

where $\lambda, \kappa = 0, \pm 1$. For $j_1 = j_3 = \frac{1}{2}$

$$4\,d\sigma_0 K_\lambda^\kappa = \delta_{\lambda,-\kappa}\frac{3}{2\pi}\sum_{\substack{S\,S'\,J\,J'\\ L\,L'\,J_0}}\sqrt{(2L+1)\,(2L'+1)}\times$$
$$\sqrt{(2S+1)\,(2S'+1)}\,(2J+1)\,(2J'+1)$$
$$(-1)^{j_2+j_4+S'+J'+L}(1\,-\lambda\,1\,\lambda|\,J_0\,0)(L'\,0\,L\,0\,|\,J_0 0)$$
$$\begin{Bmatrix} \frac{1}{2} & j_2 & S \\ S' & 1 & \frac{1}{2}\end{Bmatrix}\begin{Bmatrix} \frac{1}{2} & j_4 & J' \\ J & 1 & \frac{1}{2}\end{Bmatrix}\begin{Bmatrix} J & S & L \\ J' & S' & L' \\ 1 & 1 & J_0\end{Bmatrix}\underbrace{a_J^{L\,S}(a_{J'}^{L'\,S'})^*}$$

$$K_{+1}^{-1} = K_{-1}^{+1} = -K_x^x = -K_y^y$$

$$K_i^j = 0 , \quad i \neq j$$

$\underline{S = S' = 0}$

$$\begin{Bmatrix} \frac{1}{2} & j_2 & 0 \\ 0 & 1 & \frac{1}{2} \end{Bmatrix} = 0 \Rightarrow \text{all } K_i^j = 0$$

$\underline{S = S' = 1}$

$$K_x^x = K_y^y \neq 0$$

$$K_z^z \neq 0$$

$K_z^z \geq 0$ for $j_\theta = \frac{1}{2}$

Spin-spin correlation coefficients

$$C_{\lambda\kappa} = \frac{SpF\sigma_\lambda(1)\,\sigma_\kappa(2)\,F^+}{SpF\,F^+}, \qquad (23)$$

we found for the case of $j_1 = j_2 = \frac{1}{2}$

$$4d\sigma_0\, C_{\lambda,\kappa} = \delta_{\lambda,-\kappa}\frac{3}{2\pi}\sum_{S\,S'\,J}(-1)^{S+J}(2J+1)$$

$$\times\sqrt{(2S+1)(2S'+1)}\sum_{L\,L\,J_0}(-1)^{L'}(2J_0+1)\sqrt{2L'+1}$$

$$(1\lambda\,1-\lambda\,|\,J_0\,0)(J_0\,0\,L'\,0\,|\,L\,0)\begin{Bmatrix} S' & S & J_0 \\ L & L' & J \end{Bmatrix}\begin{Bmatrix} S' & \frac{1}{2} & \frac{1}{2} \\ S & \frac{1}{2} & \frac{1}{2} \\ J_0 & 1 & 1 \end{Bmatrix}\times$$

$$a_J^{L\,S}(a_{J'}^{L'\,S'})^*$$

$C_{+1,-1} = C_{+1,-1} = -C_{x,x} = -C_{y,y} \neq 0$, $C_0^0 = C_z^z \neq 0$ whereas $C_{i,j} = 0$, where $i \neq j$ $(i, j = x, y, z)$.

$C_{x,x} = C_{y,y} = C_{z,z} = -1$ for $S = S' = 0$.

TENSOR POLARIZED Θ^+ $(j_\Theta \geq \frac{3}{2})$

Spin-tensors for any spin j:

$T_{JM}(j)$, **J=2j, 2j-1, $\cdots$,0**

M=J, J-1, $\cdots$, -J

$$\vec{N} + N \to \vec{Y} + \vec{\Theta}^+$$

$$K^{J_Y\, M_Y, J_\Theta\, M_\Theta}_{J_N\, M_N} = \frac{Sp\left\{T_{J_Y M_Y} T_{J_\Theta M_\Theta} \hat{F}\, T_{J_N M_N}\, F^+\right\}}{SpFF^+},$$

$K^{1\, M_Y, 00}_{1\, M_N} \to K^{M_Y}_{M_N}$ **spin-transfer**

$K^{1\, M_Y, 1\, M_\Theta}_{00} \to C^{f}_{M_Y\, M_\Theta}$ **spin-spin correlation**

$K^{00, J_4\Theta\, M_\Theta}_{00} \to t_{J_\Theta\, M_\Theta}$ **tensor polarisation**

For $J_\Theta - even$, $M_\Theta = 0$: $t_{J_\Theta 0}$ is measurable by asymmetries in $\Theta^+ \to M + N$

/ S.M.Berman, M.Jacob, Phys.Rev. 139 (1965) B1023)/

$$K_{J_N\,M_N}^{J_Y\,M_Y,J_\Theta\,M_\Theta}=\delta_{J_N,0}\,\delta_{M_N,0}\,K_{00}^{J_Y M_Y,J_\Theta M_\Theta},$$

if $S=S'=0$.

for polarized beam **the final spin-tensor correlation:**

- **is absent for S=0** $pp\to\Sigma^+\Theta^+\ \pi=+1$

$pn\to\Lambda^0\Theta^+\ \pi=-1$,

- **occurs for S=1** $pp\to\Sigma^+\Theta^+\ \pi=-1$,

$pn\to\Lambda^0\Theta^+\ \pi=+1$

Example: $j_\Theta=\frac{3}{2}$, $T=0$, $\pi=+1$,

$$SpFF^+\,K_{1\,y}^{1\,y,20}=\frac{1}{4}\{|a_1^0|^2-|a_1^2|^2+$$
$$+Re(\frac{\sqrt{30}}{6}a_2^2a_1^{0^*}-\frac{\sqrt{2}}{2}a_1^2a_1^{0^*}-\frac{\sqrt{15}}{3}a_2^2a_1^{2^*})\},$$

$$SpFF^+\,t_{20}=-\frac{5}{4}|a_2^2|^2-\frac{3}{4}|a_1^2|^2-$$
$$-\frac{1}{2}Re\left(\sqrt{15}a_2^2a_1^{2^*}+\sqrt{30}a_2^2a_1^{0^*}-3\sqrt{2}a_1^2\,a_1^{0^*}\right)$$

Conclusion

Model independent analysis of the $NN \to Y\Theta^{+}$ at the threshold for arbitrary spin j_Θ $(\frac{1}{2}^{\pm}, \frac{3}{2}^{\pm}, \frac{5}{2}^{\pm})$:

$*$ **S=0**

$C_{x,x} = C_{y,y} = C_{z,z} = -1$

$K_y^y = 0$, $K_{J_N M_N}^{J_Y M_Y, J_\Theta M_\Theta} = 0$

$$\begin{cases} pp \to \Sigma^{+}\Theta^{+} & \pi = +1 \\ pn \to \Lambda^{0}\Theta^{+} & \pi = -1 \end{cases}$$

if $I_\Theta = 0$

$*$ **S=1**

$C_{x,x} = C_{y,y} \geq 0$

$K_y^y \neq 0$, $K_{J_N M_N}^{J_Y M_Y, J_\Theta M_\Theta} \neq 0$

$$\begin{cases} pp \to \Sigma^{+}\Theta^{+} & \pi = -1 \\ pn \to \Lambda^{0}\Theta^{+} & \pi = +1 \end{cases}$$

$*$ **if $j_\Theta \geq \frac{3}{2}$, then $t_{J0} \Longrightarrow \pi_\Theta$**

$*$ **The isospin T=0,1 of the NN-system acts as the P-parity π_Θ=+1, -1**

The method can be applied for others baryons

Yu. N.U. Phys. Lett. B595 (2004) 277;
hep-ph/0402216