Polarization effects in the reactions $p + {}^3He \to \pi^+ + {}^4He$, $\pi^+ + {}^4He \to p + {}^3He$ and verification of the consequences of quantum-mechanical coherence for the correlation tensor

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1. Consequences of T invariance for binary reactions

[V. V. Lyuboshitz, V. L. Lyuboshitz, Phys. At. Nucl. 63, 767 (2000)]

Direct reaction $a+b\to c+d$, inverse reaction $c+d\to a+b$.

$j_a=j_b=\frac{1}{2}$; $j_c, j_d\to$ arbitrary spins.

P-parity conservation + invariance under rotation in the 3-dimensional space + linearity of the quantum theory $\Rightarrow$ the following structure of the effective cross-section $\sigma_{a+b\to c+d}$, summed over spin projections of the final particles c and d (in the c.m. frame of (a,b)):

$$\sigma_{a+b\to c+d}(\vec{k}_a, \vec{P}^{(a)}, \vec{P}^{(b)}; \vec{k}_c) = \sigma_0(E,\Theta)\, L(\vec{k}_a, \vec{P}^{(a)}, \vec{P}^{(b)}; \vec{k}_c)$$

linear function of polarization vectors $\vec{P}^{(a)}$, $\vec{P}^{(b)}$

$$L(\vec{k}_a, \vec{P}^{(a)}, \vec{P}^{(b)}; \vec{k}_c) = 1 + A(E,\Theta)(\vec{P}^{(a)}\vec{n}) + B(E,\Theta)(\vec{P}^{(b)}\vec{n}) + C(E,\Theta)(\vec{P}^{(a)}\vec{P}^{(b)})$$
$$+ D(E,\Theta)(\vec{P}^{(a)}\vec{\ell})(\vec{P}^{(b)}\vec{\ell}) + F(E,\Theta)(\vec{P}^{(a)}\vec{m})(\vec{P}^{(b)}\vec{m}) + G(E,\Theta)(\vec{P}^{(a)}\vec{\ell})(\vec{P}^{(b)}\vec{m}) +$$
$$+ H(E,\Theta)(\vec{P}^{(a)}\vec{m})(\vec{P}^{(b)}\vec{\ell});$$

here $\vec{\ell}, \vec{m}, \vec{n}\to$ mutually orthogonal unit vectors.

$$\vec{\ell} = \vec{k}_a/k_a,\quad \vec{m} = \frac{\vec{\ell}' - \vec{\ell}(\vec{\ell}\vec{\ell}')}{\sin\Theta},\quad \vec{n} = \frac{\vec{\ell}\times\vec{\ell}'}{\sin\Theta}$$

$\left(\vec{\ell}' = \vec{k}_c/k_c,\ \Theta = \arccos(\vec{\ell}\vec{\ell}')\to \to\text{emission angle}\right).$

$\sigma_0(E,\Theta)\to$ cross-section of the reaction $a+b\to c+d$ for unpolarized particles a and b.

Inverse reaction $c+d \to a+b$; particles c and d are unpolarized; polarization vectors of the final particles $\vec{\xi}^{(a)}$ and $\vec{\xi}^{(b)}$ are fixed.

T invariance, principle of detailed balance $\Rightarrow$

$$\sigma_{c+d\to a+b} = \frac{k_a^2}{k_c^2(2j_c+1)(2j_d+1)}\,\sigma_{a+b\to c+d}\left(-\vec{k}_a, -\vec{\xi}^{(a)}, -\vec{\xi}^{(b)}; -\vec{k}_c\right)$$

(momentum directions and spin projections are replaced by opposite ones: $\vec{P}^{(a)} \to -\vec{\xi}^{(a)}$, $\vec{P}^{(b)} \to -\vec{\xi}^{(b)}$).

$$\Rightarrow \sigma_{c+d\to a+b}\left(\vec{k}_c; \vec{k}_a, \vec{\xi}^{(a)}, \vec{\xi}^{(b)}\right) = \frac{1}{4}\tilde{\sigma}_0(E,\theta)\,L\left(-\vec{k}_a, -\vec{\xi}^{(a)}, -\vec{\xi}^{(b)}; -\vec{k}_c\right),$$

where $$\tilde{\sigma}_0(E,\theta) = \frac{4k_a^2}{k_c^2(2j_c+1)(2j_d+1)}\,\sigma_0(E,\theta) \to$$

$\to$ cross-section of the inverse reaction $c+d\to a+b$ with unpolarized particles c and d, summed over the spin projections of the final particles a, b.

$\vec{\xi}^{(a)}$, $\vec{\xi}^{(b)}$ may take any values not exceeding unity in modulus: $|\vec{\xi}^{(a)}| \leq 1$, $|\vec{\xi}^{(b)}| \leq 1$.

Two-particle density matrix for the final particles $a, b \to$ can be obtained on the basis of the function L by replacing $\vec{P}^{(a)} \to -\vec{\xi}^{(a)} \to -\hat{\vec{\sigma}}^{(a)}$, $\vec{P}^{(b)} \to -\vec{\xi}^{(b)} \to -\hat{\vec{\sigma}}^{(b)}$; $\hat{\vec{\sigma}}^{(a)}$, $\hat{\vec{\sigma}}^{(b)} \to$ vector Pauli operators:

$$\hat{\rho}^{(a,b)} = \frac{1}{4}\Big[\hat{I}^{(a)} \otimes \hat{I}^{(b)} + (\vec{P}^{(a)}(E,\Theta)\hat{\vec{\sigma}}^{(a)}) \otimes \hat{I}^{(b)} + \hat{I}^{(a)} \otimes (\vec{P}^{(b)}(E,\Theta)\hat{\vec{\sigma}}^{(b)}) + \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik}(E,\Theta)\hat{\sigma}_i^{(a)} \otimes \hat{\sigma}_k^{(b)}\Big]$$

$$= \frac{1}{4}\hat{L}(-\vec{k}_a, -\hat{\vec{\sigma}}^{(a)}, -\hat{\vec{\sigma}}^{(b)}; -\vec{k}_c) \qquad (tr_{(a,b)}\hat{\rho}^{(a,b)} = 1).$$

Here $\hat{I}^{(a)}$, $\hat{I}^{(b)}$ → two-row unit matrices;

$$\left.\begin{aligned} \vec{P}^{(a)}(E,\Theta) &= -A(E,\Theta)\vec{n} \\ \vec{P}^{(b)}(E,\Theta) &= -B(E,\Theta)\vec{n} \end{aligned}\right\} \rightarrow$$ polarization vectors of the final particles a, b;

$$T_{ik}(E,\Theta) = C(E,\Theta)\delta_{ik} + D(E,\Theta)l_i l_k + F(E,\Theta)m_i m_k + G(E,\Theta)l_i m_k + H(E,\Theta)m_i l_k \rightarrow$$

components of the correlation tensor describing spin correlations in the two-particle system (a, b).

All the functions $A, B, C, D, F, G, H, \sigma_0$ → the same functions of energy E and emission angle Θ as for the direct reaction $a+b \rightarrow c+d$, the unit vectors $\vec{l}, \vec{m}, \vec{n}$ → also the same as before.

Thus, due to T invariance, the dependence of the effective cross-section for the direct reaction $a+b \rightarrow c+d$ upon the polarizations of initial particles completely determines the polarization vectors and spin correlations for the same particles a, b produced in the inverse reaction $c+d \rightarrow a+b$ with unpolarized primary particles.

2) Polarization effects in the reaction $p + {}^3He \to \pi^+ + {}^4He$.

This reaction $\Rightarrow$ of the type $\frac{1}{2} + \frac{1}{2} \to 0 + 0$ (proton and ${}^3He$ have spin $\frac{1}{2}$, π^+ and ${}^4He$ have zero spin).

Thus, as follows from the parity and angular-momentum conservation (taking into account the negative internal parity of the $\pi^\pm$-meson), this reaction is possible only for triplet states of the system $(p, {}^3He)$.

Axis of total-spin quantization $z \to$ along $\vec{\ell} = \vec{k}_p / k_p$;

$x, y \to$ along the directions of $\vec{m}, \vec{n}$ with $\vec{\ell}' = \vec{k}_\pi / k_\pi$.

Three possible triplet states of the $(p, {}^3He)$-system (spin projections $+1$, -1, 0 onto the axis z):

$$|+1, \vec{\ell}\rangle = |+\tfrac{1}{2}, \vec{\ell}\rangle^{(p)} \otimes |+\tfrac{1}{2}, \vec{\ell}\rangle^{(He)}$$

$$|-1, \vec{\ell}\rangle = |-\tfrac{1}{2}, \vec{\ell}\rangle^{(p)} \otimes |-\tfrac{1}{2}, \vec{\ell}\rangle^{(He)}$$

$$|0, \vec{\ell}\rangle = \frac{1}{\sqrt{2}} \left(|+\tfrac{1}{2}, \vec{\ell}\rangle^{(p)} \otimes |-\tfrac{1}{2}, \vec{\ell}\rangle^{(He)} + |-\tfrac{1}{2}, \vec{\ell}\rangle^{(p)} \otimes |+\tfrac{1}{2}, \vec{\ell}\rangle^{(He)} \right)$$

Two-particle spin density matrix for the initial state:

$$\hat{\rho}^{(p, He)} = \frac{1}{4} \left(\hat{I}^{(p)} + \vec{P}^{(p)} \hat{\vec{\sigma}}^{(p)} \right) \otimes \left(\hat{I}^{(He)} + \vec{P}^{(He)} \hat{\vec{\sigma}}^{(He)} \right)$$

($\vec{P}^{(p)}$, $\vec{P}^{(He)}$ $\to$ independent polarization vectors)

Helicity amplitude $R_\lambda(E,\Theta) \to$ amplitude of the reaction $p+{}^3He \to \pi^+ + {}^4He$ proceeding from the state $|\lambda, \vec{\ell}\rangle$ ($\lambda = \pm 1, 0$)

($R_\lambda(E,\Theta) = \sum_J (2J+1)\, \gamma^{(J)}(E)\, d^{(J)}_{0\lambda}(\Theta) \Rightarrow$ expansion over total angular momenta; $d^{(J)}_{0\lambda}(\Theta)$ – Wigner functions)

$\Rightarrow$ the differential cross-section for this reaction acquires the form:

$$\sigma_{p+{}^3He \to \pi^+ + {}^4He}(\vec{k}_p, \vec{P}^{(p)}, \vec{P}^{(He)}; \vec{k}_\pi) = \langle \psi | \hat{\rho}^{(p, He)} | \psi \rangle =$$

$$= \sum_\lambda \sum_{\lambda'} R_\lambda(E,\Theta) \langle \lambda, \vec{\ell} | \hat{\rho}^{(p,He)} | \lambda', \vec{\ell} \rangle R^*_{\lambda'}(E,\Theta) .$$

($|\psi\rangle = \sum_{\lambda=\pm 1, 0} R^*_\lambda(E,\Theta) |\lambda, \vec{\ell}\rangle \to$ means the unnormalized initial two-particle spin state selected by the reaction

as follows from parity conservation:

$$R_\lambda(E,\Theta) = (-1)^{|\lambda|} R_{-\lambda}(E,\Theta) \Rightarrow R_{+1}(E,\Theta) = -R_{-1}(E,\Theta) \equiv R_1(E,\Theta)$$

$$\Rightarrow |\psi\rangle = \left(R^*_1(E,\Theta) - \frac{i}{\sqrt{2}} R^*_0(E,\Theta)\right) |+1, n\rangle +$$
$$+ \left(R^*_1(E,\Theta) + \frac{i}{\sqrt{2}} R^*_0(E,\Theta)\right) |-1, n\rangle ;$$

here $|\lambda, \vec{n}\rangle \to$ states with the total-spin projections λ to the normal to the reaction plane, representing mutually orthogonal superpositions of triplet states $|\lambda, \vec{\ell}\rangle$:

$$|+1,\vec{n}\rangle = \frac{1}{2}|+1,\vec{\ell}\rangle + \frac{i}{2}|0,\vec{\ell}\rangle - \frac{1}{2}|-1,\vec{\ell}\rangle,$$

$$|-1,\vec{n}\rangle = \frac{1}{2}|+1,\vec{\ell}\rangle - \frac{i}{2}|0,\vec{\ell}\rangle - \frac{1}{2}|-1,\vec{\ell}\rangle,$$

$$|0,\vec{n}\rangle = \frac{i}{2}\left(|+1,\vec{\ell}\rangle + |-1,\vec{\ell}\rangle\right) \rightarrow \text{„forbidden" state};$$

the reaction can proceed only from the states $|+1,\vec{n}\rangle$ and $|-1,\vec{n}\rangle$ of the initial system $(p, {}^3He)$.

Calculation of the differential cross-section $\sigma_{p+{}^3He\to\pi^++{}^4He}$.

$$|\psi\rangle = R_1^*(E,\theta)\left(|+\tfrac{1}{2},z\rangle^{(p)}\otimes|+\tfrac{1}{2},z\rangle^{(He)} - |-\tfrac{1}{2},z\rangle^{(p)}\otimes|-\tfrac{1}{2},z\rangle^{(He)}\right) + \frac{1}{\sqrt{2}}R_0^*(E,\theta)\left(|+\tfrac{1}{2},z\rangle^{(p)}\otimes|-\tfrac{1}{2},z\rangle^{(He)} + |-\tfrac{1}{2},z\rangle^{(p)}\otimes|+\tfrac{1}{2},z\rangle^{(He)}\right)$$

(here $z \parallel \vec{\ell}$);

$$\sigma_{p+{}^3He\to\pi^++{}^4He} = \langle\psi|\hat{\rho}^{(p,He)}|\psi\rangle =$$

$$= \frac{1}{4}\langle\psi|\left(\hat{I}^{(p)} + \vec{P}^{(p)}\hat{\vec{\sigma}}^{(p)}\right)\otimes\left(\hat{I}^{(He)} + \vec{P}^{(He)}\hat{\vec{\sigma}}^{(He)}\right)|\psi\rangle$$

$\Rightarrow$ expectation values for the tensor products of Pauli matrices, incorporated in $\hat{\rho}^{(p,He)}$ (for information); here $\hat{\sigma}_z = \hat{\vec{\sigma}}\vec{\ell}$, $\hat{\sigma}_x = \hat{\vec{\sigma}}\vec{m}$, $\hat{\sigma}_y = \hat{\vec{\sigma}}\vec{n}$:

$$\langle\psi|\hat{\sigma}_y^{(P)}\otimes\hat{I}^{(He)}|\psi\rangle=\langle\psi|\hat{I}^{(P)}\otimes\hat{\sigma}_y^{(He)}|\psi\rangle=2\sqrt{2}\,\mathrm{Im}\left(R_1(E,\Theta)R_0^*(E,\Theta)\right)$$

$$\langle\psi|\hat{\sigma}_z^{(P)}\otimes\hat{\sigma}_x^{(He)}|\psi\rangle=\langle\psi|\hat{\sigma}_x^{(P)}\otimes\hat{\sigma}_z^{(He)}|\psi\rangle=2\sqrt{2}\,\mathrm{Re}\left(R_1(E,\Theta)R_0^*(E,\Theta)\right)$$

$$\langle\psi|\hat{\sigma}_z^{(P)}\otimes\hat{\sigma}_z^{(He)}|\psi\rangle=2|R_1(E,\Theta)|^2-|R_0(E,\Theta)|^2;$$

$$\langle\psi|\hat{\sigma}_x^{(P)}\otimes\hat{\sigma}_x^{(He)}|\psi\rangle=-2|R_1(E,\Theta)|^2+|R_0(E,\Theta)|^2;$$

$$\langle\psi|\hat{\sigma}_y^{(P)}\otimes\hat{\sigma}_y^{(He)}|\psi\rangle=2|R_1(E,\Theta)|^2+|R_0(E,\Theta)|^2;$$

$$\langle\psi|\hat{\sigma}_z^{(P)}\otimes\hat{I}^{(He)}|\psi\rangle=\langle\psi|\hat{I}^{(P)}\otimes\hat{\sigma}_z^{(He)}|\psi\rangle=\langle\psi|\hat{\sigma}_x^{(P)}\otimes\hat{I}^{(He)}|\psi\rangle=$$
$$=\langle\psi|\hat{I}^{(P)}\otimes\hat{\sigma}_x^{(He)}|\psi\rangle=\langle\psi|\hat{\sigma}_x^{(P)}\otimes\hat{\sigma}_y^{(He)}|\psi\rangle=\langle\psi|\hat{\sigma}_y^{(P)}\otimes\hat{\sigma}_x^{(He)}|\psi\rangle=$$
$$=\langle\psi|\hat{\sigma}_y^{(P)}\otimes\hat{\sigma}_z^{(He)}|\psi\rangle=\langle\psi|\hat{\sigma}_z^{(P)}\otimes\hat{\sigma}_y^{(He)}|\psi\rangle=0$$

$\Rightarrow$ finally, $\sigma_{p+{}^3He\to\pi^++{}^4He}$ is described by the general structural formula for $\sigma_{a+b\to c+d}$, where the functions $\sigma_0, A, B, C, D, F, G, H$ are bilinear combinations of the helicity amplitudes R_1, R_0:

$$\sigma_0(E,\Theta)=\frac{1}{4}\langle\psi|\psi\rangle=\frac{1}{4}\left(|R_0(E,\Theta)|^2+2|R_1(E,\Theta)|^2\right);$$

$$A(E,\Theta)=B(E,\Theta)=\frac{1}{\sqrt{2}\,\sigma_0(E,\Theta)}\mathrm{Im}\left(R_1(E,\Theta)R_0^*(E,\Theta)\right);$$

$$C(E,\Theta)=1; \qquad D(E,\Theta)=-\frac{|R_0(E,\Theta)|^2}{2\sigma_0(E,\Theta)}; \quad F(E,\Theta)=-\frac{|R_1(E,\Theta)|^2}{2\sigma_0(E,\Theta)}$$

$$G(E,\Theta)=H(E,\Theta)=\frac{1}{\sqrt{2}\,\sigma_0(E,\Theta)}\mathrm{Re}\left(R_1(E,\Theta)R_0^*(E,\Theta)\right)$$

Particular case:

$(\Theta = 0;\ \Theta = \pi) \Rightarrow R_1(E,\Theta) \to 0$ (due to the conservation of the angular-momentum projection onto the reaction axis) $\Rightarrow$ the considerably simpler form for the dependence of $\sigma_{p+{}^3He\to\pi^+ + {}^4He}$ upon the proton and ${}^3He$ polarization vectors:

$$\sigma_{p+{}^3He\to\pi^+ + {}^4He} = \frac{1}{4}|R_0|^2\left(1 + \vec{P}^{(p)}\vec{P}^{(He)} - 2(\vec{P}^{(p)}\vec{\ell})(\vec{P}^{(He)}\vec{\ell})\right)$$

(here the coefficient at $|R_0|^2$ reflects the fraction of the state $|0,\vec{\ell}\rangle$ with zero projection of the total spin in the initial states).

Integration of the general expression for $\sigma_{p+{}^3He\to\pi^+ + {}^4He}$ over the azimuthal angle $\Rightarrow$

$$\sigma_{p+{}^3He\to\pi^+ + {}^4He}(E,\Theta,\vec{P}^{(p)},\vec{P}^{(He)}) =$$
$$= \frac{\pi}{2}|R_0(E,\Theta)|^2\left(1 + \vec{P}^{(p)}\vec{P}^{(He)} - 2(\vec{P}^{(p)}\vec{\ell})(\vec{P}^{(He)}\vec{\ell})\right) +$$
$$+ \pi|R_1(E,\Theta)|^2\left(1 + (\vec{P}^{(p)}\vec{\ell})(\vec{P}^{(He)}\vec{\ell})\right)$$

(the terms with $C(E,\Theta)$, $D(E,\Theta)$ and $F(E,\Theta)$ are retained, and the terms with $A(E,\Theta)$, $B(E,\Theta)$, $G(E,\Theta)$, $H(E,\Theta)$ vanish).

3) Spin effects in the inverse reaction $\pi^+ + {}^4He \to p + {}^3He$.

The $(p, {}^3He)$-system is produced in the triplet state only.
This specific triplet state is symmetric with respect to the interchange of spin quantum numbers of p and ${}^3He$.
Form of this state (normalized to unity):

$$|\tilde{\psi}\rangle = \frac{1}{\left(|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2\right)^{1/2}} \Big[R_1(E,\theta)\Big(|+1/2, \vec{e}\rangle^{(p)} \otimes |+1/2, \vec{e}\rangle^{(He)} - |-1/2, \vec{e}\rangle^{(p)} \otimes |-1/2, \vec{e}\rangle^{(He)} \Big) + \frac{1}{\sqrt{2}} R_0(E,\theta) \Big(|+1/2, \vec{e}\rangle^{(p)} \otimes |-1/2, \vec{e}\rangle^{(He)} + |-1/2, \vec{e}\rangle^{(p)} \otimes |+1/2, \vec{e}\rangle^{(He)} \Big) \Big] =$$

$$= \frac{1}{\left(|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2\right)^{1/2}} \Big[\Big(R_1(E,\theta) - \frac{i}{\sqrt{2}} R_0(E,\theta) \Big) |+1, \vec{n}\rangle + \Big(R_1(E,\theta) + \frac{i}{\sqrt{2}} R_0(E,\theta) \Big) |-1, \vec{n}\rangle \Big]$$

(as before, $|+1, \vec{n}\rangle$, $|-1, \vec{n}\rangle \to$ states with the total-spin projections $+1, -1$ onto the normal to the reaction plane)

Final triplet state $|\tilde{\psi}\rangle$ in the time-inverted process $\pi^+ + {}^4He \to p + {}^3He$ => similar in structure to the initial triplet state $|\psi\rangle$ selected by the direct reaction $p + {}^3He \to \pi^+ + {}^4He$, but differs by complex conjugation of helicity amplitudes.

Following the general relations originating from T invariance:

1) Effective cross-section for the reaction $\pi^+ + {}^4He \to p + {}^3He$ in the c.m. frame, summed over the spin projections in the final state:

$$\tilde{\sigma}_0(E,\Theta) = \left(k_p / k_\pi\right)^2 \left(|R_0(E,\Theta)|^2 + 2|R_1(E,\Theta)|^2\right).$$

2) Polarizations vectors of the proton and ${}^3He$ in the final system:

$$\vec{P}^{(p)}(E,\Theta) = \langle\tilde{\psi}|\hat{\vec{\sigma}}^{(p)}|\tilde{\psi}\rangle = \vec{P}^{(He)}(E,\Theta) = \langle\tilde{\psi}|\hat{\vec{\sigma}}^{(He)}|\tilde{\psi}\rangle =$$

$$= -A(E,\Theta)\vec{n} = -2\sqrt{2}\,\frac{\mathrm{Im}\left(R_1(E,\Theta)R_0^*(E,\Theta)\right)}{|R_0(E,\Theta)|^2 + 2|R_1(E,\Theta)|^2}\,\vec{n}.$$

3) Correlation tensor for the $(p, {}^3He)$-system:

$$T_{ik}(E,\Theta) = \langle\tilde{\psi}|\hat{\sigma}_i^{(p)} \otimes \hat{\sigma}_k^{(He)}|\tilde{\psi}\rangle =$$

$$= \delta_{ik} - \frac{2}{|R_0(E,\Theta)|^2 + 2|R_1(E,\Theta)|^2}\Big\{|R_0(E,\Theta)|^2 l_i l_k + 2|R_1(E,\Theta)|^2 m_i m_k -$$

$$- \sqrt{2}\,\mathrm{Re}\left(R_1(E,\Theta)R_0^*(E,\Theta)\right)(l_i m_k + m_i l_k)\Big\}$$

In all these expressions: the helicity amplitudes $R_0(E,\Theta)$, $R_1(E,\Theta)$ and the unit vectors $\vec{\ell}, \vec{m}, \vec{n}$ are the same as for the previously considered direct process $p + {}^3He \to \pi^+ + {}^4He$.

One can see, in particular, that:

1) polarization of the ${}^3He$ nucleus along the normal to the reaction plane is identical to that of the proton;

2) the correlation tensor $T_{ik}(E,\Theta)$, describing the spin correlations in the $(p, {}^3He)$-system, is symmetric.

Thus, the spins of the proton and the ${}^3He$ nucleus in the reaction $\pi^+ + {}^4He \to p + {}^3He$ must be tightly correlated.

$\Rightarrow$ a possibility arises to prepare a beam of ${}^3He$ nuclei with controllable polarization without acting directly on these nuclei.

Example: let a proton be produced in the reaction $\pi^+ + {}^4He \to p + {}^3He$ and then be scattered on a spinless or an unpolarized target (e.g. on a ${}^{12}C$ nucleus); the analyzing power is characterized by the vector $\vec{\xi}^{(p)} = \alpha_p(k_p, \Theta_p)\,\vec{t}^{(p)}$ ($\Theta_p \to$ secondary scattering angle; $\vec{t}^{(p)} \to$ unit vector along the normal to the scattering plane.)

Then the components of the polarization vector of the unscattered ^{3}He nucleus are as follows (dependence on $\vec{\xi}^{(p)}$):

$$\tilde{P}_k^{(He)}(\vec{\xi}^{(p)}) = \frac{P_k^{(He)} + \sum_{i=1}^{3} T_{ik}(E,\Theta)\xi_i^{(p)}}{1 + \vec{P}^{(p)}(E,\Theta)\vec{\xi}^{(p)}} .$$

(Here: $\vec{P}^{(p)}$, $\vec{P}^{(He)}$ → polarization vectors of the proton and ^{3}He in the absence of secondary scattering;
$T_{ik}(E,\Theta)$ → correlation tensor for the $(p, {}^3He)$-system).

Particular case: $\Theta = 0$ $(R_1(E,\Theta) = 0)$ $\Rightarrow \begin{cases} \vec{P}^{(p)} = \vec{P}^{(He)} = 0 \\ T_{ik}(E,0) = \delta_{ik} - 2\ell_i \ell_k \end{cases}$

$\Rightarrow$ at the proton scattering on the ^{12}C nucleus, the observed polarization of ^{3}He is:

$$\tilde{\vec{P}}^{(He)}(\vec{\xi}^{(p)}) = \alpha_p(E_p, \Theta_p)\left(\vec{t}^{(p)} - 2\vec{\ell}(\vec{\ell}\vec{t}^{(p)})\right);$$

$|\tilde{\vec{P}}^{(He)}| = |\alpha_p(E_p, \Theta_p)| \Rightarrow$ maximum spin correlation.

Correlations at secondary scattering:

[V. L. Lyuboshitz, M. I. Podgoretsky. Phys. At. Nucl. 60, 39 (1997)]

4. Violation of „classical" incoherence inequalities for the correlation tensor.

For incoherent mixtures of factorizable two-particle states of fermions with spin 1/2, the following inequalities for the components of the correlation tensor should be satisfied:

$$|T_{11}+T_{22}+T_{33}| \leqslant 1; \quad |T_{11}+T_{22}| \leqslant 1;$$

$$|T_{11}+T_{33}| \leqslant 1; \quad |T_{22}+T_{33}| \leqslant 1$$

[R. Lednicky, V. L. Lyuboshitz, Phys. Lett. B508, 146 (2001)

However: these inequalities may be violated for non-factorizable quantum-mechanical superpositions.

The triplet state of the final system $(p, {}^3He)$ in the reaction $\pi^+ + {}^4He \to p + {}^3He \Rightarrow$ characteristic example of such non-factorizable spin states:

$$|\tilde{\Psi}\rangle = \frac{1}{\left(|R_0(E,\Theta)|^2 + 2|R_1(E,\Theta)|^2\right)^{1/2}} \Big[R_1(E,\Theta)\left(|+\tfrac{1}{2},\vec{\ell}\rangle^{(p)} \otimes |+\tfrac{1}{2},\vec{\ell}\rangle^{(He)} - |-\tfrac{1}{2},\vec{\ell}\rangle^{(p)} \otimes |-\tfrac{1}{2},\vec{\ell}\rangle^{(He)}\right) + \frac{1}{\sqrt{2}} R_0(E,\Theta)\left(|+\tfrac{1}{2},\vec{\ell}\rangle^{(p)} \otimes |-\tfrac{1}{2},\vec{\ell}\rangle^{(He)} + |-\tfrac{1}{2},\vec{\ell}\rangle^{(p)} \otimes |+\tfrac{1}{2},\vec{\ell}\rangle^{(He)}\right) \Big]$$

Calculation of components of the correlation tensor for $(p, {}^3He)$-system (in the frame with $z \parallel \vec{\ell}$, $x \parallel \vec{m}$, $y \parallel \vec{n}$) $\Longrightarrow$ the following expressions $\left(\text{indexes: } \begin{matrix} 1 \to x \\ 2 \to y \\ 3 \to z \end{matrix}\right)$:

$$T_{11} = \frac{|R_0|^2 - 2|R_1|^2}{|R_0|^2 + 2|R_1|^2};$$

$$(tr\, T = 1)$$

$$T_{22} = 1; \quad T_{33} = \frac{2|R_1|^2 - |R_0|^2}{2|R_1|^2 + |R_0|^2} = -T_{11};$$

$$T_{13} = T_{31} = \frac{2\sqrt{2}}{|R_0|^2 + 2|R_1|^2} Re\,(R_1 R_0^*); \quad T_{23} = T_{32} = T_{12} = T_{21} = 0;$$

$\Longrightarrow$ it is well seen that, indeed, one of the incoherence inequalities is obligatorily violated in the reaction $\pi^+ + {}^4He \to p + {}^3He$, irrespective of the concrete mechanism of generation of the $(p, {}^3He)$-system

$$\left(\begin{matrix} \text{at } |R_0|^2 > 2|R_1|^2 \Rightarrow |T_{11} + T_{22}| > 1 \\ \text{at } |R_0|^2 < 2|R_1|^2 \Rightarrow |T_{22} + T_{33}| > 1 \end{matrix}\right.$$

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5 Summary

1. It is shown that, on account of T invariance, the dependence of the effective cross-section for a direct binary reaction $a+b \to c+d$ upon the polarization vectors of initial particles a, b determines completely the polarization vectors and spin correlations for the same particles a, b in the inverse reaction $c+d \to a+b$ with unpolarized primary particles.

2. Using the formalism of helicity amplitudes, polarization effects have been studied in the reaction $p+{}^3He \to \pi^+ + {}^4He$ and in the inverse process $\pi^+ + {}^4He \to p + {}^3He$. It is established that, in the reaction $\pi^+ + {}^4He \to p + {}^3He$, the spins of the proton and ${}^3He$ nucleus are strongly correlated. A structural expression through the helicity amplitudes, corresponding to arbitrary angles of emission of the $(p, {}^3He)$-system, is obtained for the correlation tensor.

3. It is demonstrated that, in the reaction $\pi^+ + {}^4He \to p + {}^3He$, one of the „classical" incoherence inequalities for components of the correlation tensor is obligatorily violated.