Rotating Black Hole as Spinning Particle

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Abstract

The Kerr spinning particle has a remarkable analytical twistorial structure. Analyzing electromagnetic excitations of the Kerr circular string which are aligned to this structure, we obtain a simple stringy skeleton of the spinning particle which is formed by a topological coupling of the Kerr circular singular string and by an axial singular stringy system.

We show that the chiral traveling waves, related to an orientifold world-sheet of the axial stringy system, are described by the massive Dirac equation, so we argue that the axial string may play the part of a stringy carrier of wave function and play also a dominant role in the scattering processes.

A key role of the third, *complex* Kerr string is discussed. We conjecture that it may be one more alternative to the Witten twistor string, and a relation to the spinor helicity formalism is also discussed.

1 Before Introduction. Twistors.

In the recent paper [14] Witten suggested a string model with a twistorial target space and argued that some remarkable analytical properties of the perturbative scattering amplitudes in Yang-Mills theory [15] have the origin in an holomorphic structure of this string resulting to the holomorphy of the maximally helicity violating (MHV) amplitudes [16, 17].

Twistor boom 2004. More 55 works for 9 months!!!

For the massless and relativistic particles with spin, the external line factors of the amplitudes are described in the **Spinor Helicity formalism** which is based on a color decomposition and a reduced description in terms of the lightlike momentum p^μ and $\pm$ helicities.

The lightlike momentum $p_{\alpha\dot{\alpha}}$ is represented via the spinor ψ_α and Pauli matricies σ^μ: $p^\mu = \bar{\psi}\sigma^\mu\psi$. Geometrically, it is a light ray. It may be extended to a complex plane if $\bar{\psi}$ is independent from ψ. This plane is spanned by vector p^μ end extra complex null vector of polarization. It shows that null vector p^μ may be described only by the undoted spinor

$< p| = p^\alpha, \quad |p >= p_\alpha,$

or only by doted spinors

$[p| = p^{\dot{\alpha}}, \quad |p] = p_{\dot{\alpha}}$

in the form $p = \pm|p > [p|$.

Twistors describe the lightlike particles which are going aside of the point of observation and care an extra angular momentum. This case has to be described by extra spinor parameter $\omega_{\dot{\alpha}} = x^\nu \sigma_{\nu\dot{\alpha}\alpha}\mu^\alpha$ which fixes the position (equation) of the ray. Twistor is represented by two spinors $Z^a = \{\psi^\alpha, \omega_{\dot{\alpha}}\}$.

2 Black Hole which is neither Black nor Hole.

The Kerr geometry has found application in a very wide range of physical systems: from the rotating black holes and galactic nucleus to fundamental solutions of the low energy string theory. It displays also some remarkable relations to the structure of spinning particles. Angular momentum of the spinning particles is very high and the black-hole horizons disappear.

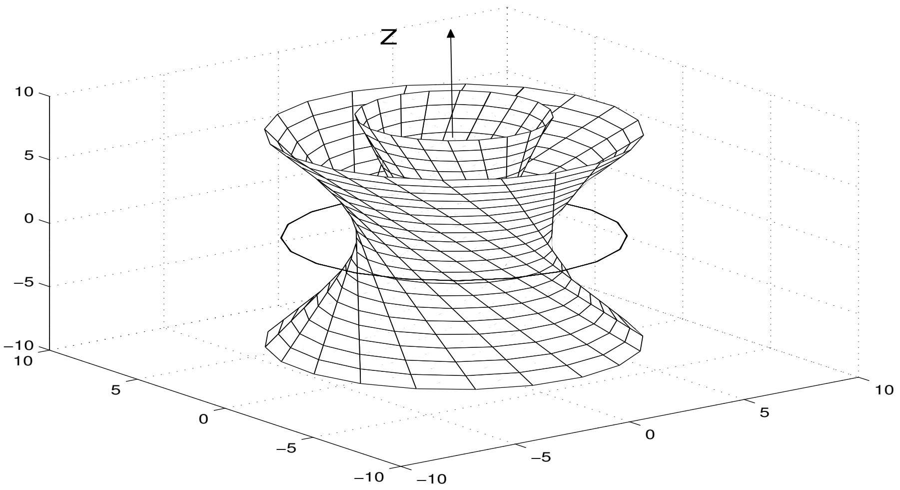

Figure 1: The Kerr circular string and null rays of the PNC which propagate from 'negative' sheet of space-time onto 'positive' one.

The Kerr singular ring was considered as a gravitational waveguide caring the traveling electromagnetic waves which generate the spin and mass of the Kerr spinning particle forming a microgeon with spin [3]. The Kerr ring represents a closed string, and the traveling waves are the string excitations. It was shown [6] that the field around this ring is similar to the field around a heterotic string, and that the Kerr ring is a chiral D-string having an orientifold world-sheet [11].

3 Axial stringy system

Exact solutions for electromagnetic excitations of the Kerr circular string [12, 13].

Solutions have to be *aligned* to the Kerr PNC (do not spoil the analiticity of its twistorial structure). This demand takes the form $F^{\mu\nu}k_\mu = 0$. The general aligned solution is described by two self-dual tetrad components $\mathcal{F}_{12} = AZ^2$ and $\mathcal{F}_{31} = \gamma Z - (AZ)_{,1}$, where function A has the form $A = \psi(Y,\tau)/P^2$, where $P = 2^{-1/2}(1 + Y\bar{Y})$, and ψ is an arbitrary holomorphic function of τ which is a complex retarded-time parameter. Function $Y(x)$ is a projection of sphere onto complex plane. It is singular at $\theta = \pi$, and a singularity will be present in any holomorphic function $\psi(Y)$. Therefore, **all the aligned e.m. solutions turn out to be singular at some angular direction** θ. It leads to the appearance of an **extra axial stringy system.** The Kerr spinning particle acquires a stringy skeleton which is formed by a topological coupling of the Kerr circular string and the axial stringy system.

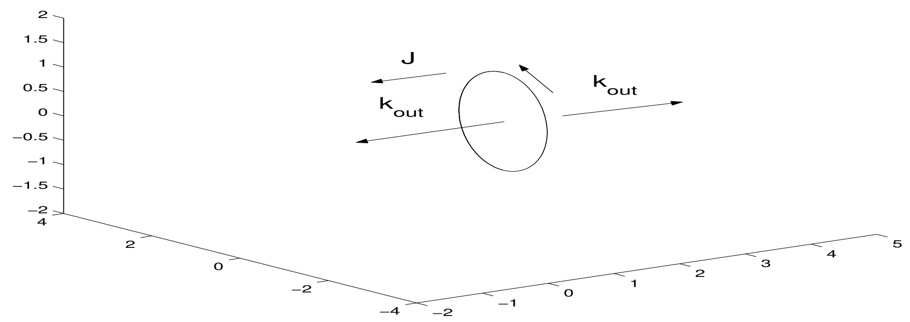

Figure 2: Stringy skeleton of the spinning particle. Circular string and two semi-infinite D-strings of opposite chiralities.

4 Dirac Equation

In the Weyl basis the Dirac current can be represented as a sum of two lightlike components of opposite chirality

$$J_\mu = e(\bar{\Psi}\gamma_\mu\Psi) = e(\chi^+\sigma_\mu\chi + \phi^+\bar{\sigma}^\mu\phi). \tag{1}$$

In this model the massive Dirac equation describes the Kerr axial stringy system - the lightlike currents of two opposite chiralities. The four-component spinor Ψ describes an interplay of the axial string currents on the different folds of the Kerr space.

Function Y as a projective spinor field $Y = \phi_2/\phi_1$. Near the z^+ semi-axis $Y \to 0$, and this spinor describes the lightlike vector $k_R = d(t-z) = \bar{\phi}_{\dot\alpha}\bar{\sigma}_\mu^{\dot\alpha\alpha}dx^\mu\phi_\alpha$. Similar, near the z^- semi-axis $\bar{Y} \to \infty$, and this limit describes the lightlike vector $k_L = d(t+z) = \bar{\chi}^\alpha\sigma_{\mu\alpha\dot\alpha}dx^\mu\chi^{\dot\alpha}$. The vectors k_L and k_R are the generators of the left and right chiral waves correspondingly. The functions ϕ and χ are fixed up to arbitrary gauge factors. Forming the four component spinor function $\Psi = \mathcal{M}(p_\mu x^\mu)\begin{pmatrix} a\phi_\alpha \\ b\chi^{\dot\alpha}\end{pmatrix}$, we obtain from the Dirac equations $am = (p_0 - p_z)b\ln\mathcal{M}'$, $\quad bm = (p_0 + p_z)a\ln\mathcal{M}'$, $p_x + ip_y = 0$, and $\mathcal{M} = e^{-i\omega t + izp_z}$.

This solution describes a wavefunction of a free spinning particle moving along the z-axis. It oscillates with the Compton frequency, which is determined by excitations of the Kerr circular string, and leads to a plane fronted modulation of the axial string by de Broglie periodicity. We are led to conclusion that the axial stringy system acquires in the Kerr spinning particle the role of *a stringy carrier of wavefunctions.*

5 Complex Kerr string

The complex Kerr string appears naturally in the initiated by Newman *complex representation of the Kerr geometry* where the Kerr source is generated by a complex world line $X_0^\mu(\tau)$.

Complex time $\tau = t + i\sigma$ consists of two parameters t and σ, it parametrizes a world sheet, and therefore, the complex world lines can be considered as strings. In the stationary Kerr case $X_0^\mu(\tau) = (\tau, 0, 0, ia)$. The real fields on the real space-time x^μ are determined via a complex retarded-time construction, where the vectors $K^\mu = x^\mu - X_0^\mu(\tau)$ have to satisfy the complex light-cone constraints $K_\mu K^\mu = 0$.

The complex retarded-time equation $t - \tau \equiv t - t_0 - i\sigma = \tilde{r}$, (where $\tilde{r} = r + ia\cos\theta$ is the Kerr complex radial distance, θ is a direction of the null ray (twistor) and r is the spatial distance along the ray), shows that on the real space-time $\sigma = -a\cos\theta$. Therefore, the light-cone constrains select a strip on τ plane, $\sigma \in [-a, +a]$, and the complex world sheet $X_0^\mu(t, \sigma)$ acquires the boundary, forming *an open complex string.*

The complex light cones, adjoined to each point of the world sheet, split into the 'right' and 'left' null planes which are the twistors having their 'origins' X_0^ν at the points of the world sheet. The 'left' null planes - twistors - form a holomorphic twistor subspace. The two twistors which are joined to the ends of the complex string, $X_0^\nu(t \pm ia)$, have the directions $\theta = 0, \pi$ and are generators of the singular $z^\pm$ semi-strings, so *the complex string turns out to be a D-string which stuck to two singular semi-strings of opposite chiralities.* Therefore, $z^\pm$ singular strings may carry the Chan-Paton factors which will play the role of quarks with respect to the complex string.

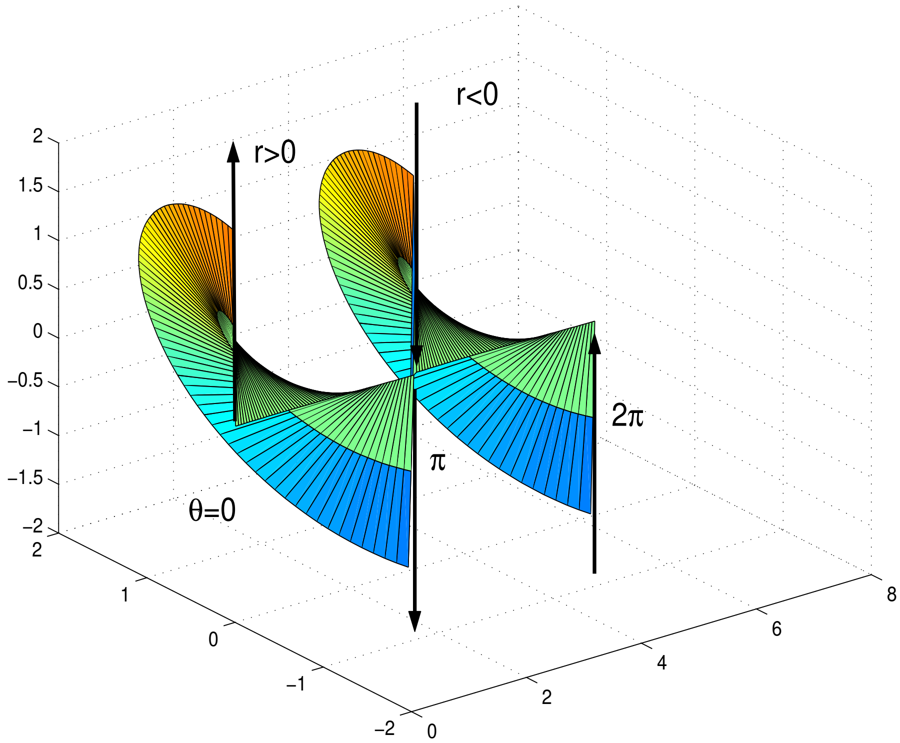

Figure 3: Complex string with adjoined twistors is stuck to two singular semi-strings of opposite chiralities. Interval is extended to 2π to the subsequent formation of orientifold.

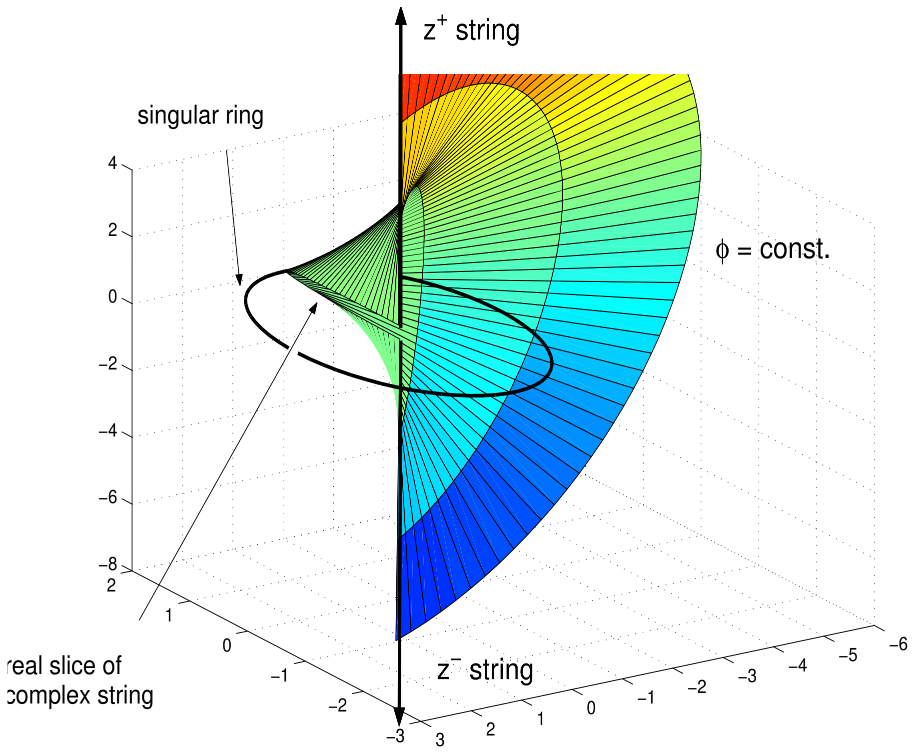

Figure 4: The complex twistor-string is imbedded into the real Kerr geometry. Thwistors generate the Kerr angular coordinate $\phi = const.$

Appendix A: Twistorial structure of the Kerr congruence

The Kerr-Schild form of the metric is $g_{\mu\nu} = \eta_{\mu\nu} + 2hk_\mu k_\nu$, where $\eta_{\mu\nu}$ is the metric of auxiliary Minkowski space-time (signature $- + ++$) in the Cartesian coordinates $x^\mu = (t, x, y, z)$, $h = \frac{mr - e^2/2}{r^2 + a^2\cos^2\theta}$, and k_μ is a twisting null field which forms the Kerr principal null congruence (PNC) - a family of the geodesic and shear free null curves.

Each null ray of the PNC represents the twistor $Z^a = \{\mu^\alpha, \omega_{\dot\alpha}\}$, in which spinor μ^α determines the null direction, $k_\nu = \bar\mu^{\dot\alpha} \sigma_{\nu\dot\alpha\alpha} \mu^\alpha$, and $\omega_{\dot\alpha} = x^\nu \sigma_{\nu\dot\alpha\alpha}\mu^\alpha$ fixes the position (equation) of the ray.

The Kerr's geodesic and shear-free PNC is determined by *the Kerr theorem* via the solution $Y(x)$ of the algebraic equation $F=0$, where $F(Y,\lambda_1,\lambda_2)$ is an arbitrary holomorphic function of the projective twistor coordinates $\{Y, \quad \lambda_1 = \zeta - Yv, \quad \lambda_2 = u + Y\bar\zeta\} = Z^a/Z^0$, and

$$
\begin{aligned}
2^{\frac12}\zeta = x+iy, \quad 2^{\frac12}\bar\zeta &= x - iy,\\
2^{\frac12}u = z-t, \quad 2^{\frac12}v &= z+t
\end{aligned}
\tag{2}
$$

are the null Cartesian coordinates.

In the Kerr-Schild formalism the projective spinor field $Y(x) = \mu^2/\mu^1$ determines the null field $k^\mu(x)$ and other parameters of the solution. In particular, the Kerr-Schild null tetrad is given by

$$
\begin{aligned}
e^1 &= d\zeta - Ydv, \qquad e^2 = d\bar\zeta - \bar Y dv,\\
e^3 &= du + \bar Y d\zeta + Y d\bar\zeta - Y\bar Y dv,\\
e^4 &= dv + he^3,
\end{aligned}
\tag{3}
$$

and the system of equations $F(Y)=0; \quad dF(Y)/dY = 0$ determines the singular lines which are caustics of the PNC.

In the Kerr solution function F is quadratic in Y, and the equation $F=0$ and the system $F(Y) = dF(Y)/dY = 0$ can be explicitly resolved [23] yielding the structure of the Kerr PNC and the Kerr singular ring shown in the Fig.2.

The Kerr singular ring is the branch line of the Kerr space on two sheets: 'positive' ($r>0$) and 'negative' ($r<0$) ones, so the Kerr PNC propagates from the 'negative' sheet onto 'positive' one through the disk spanned by the Kerr ring. Since the PNC determines a flow of radiation (in radiative solutions), one sees that outgoing radiation is compensated by an ingoing flow on the negative sheet, so the negative sheet acquires the interpretation as a list of advanced fields which are related to the vacuum zero point fields [11, 23, 12]. In this interpretation, the wave excitations can be treated as a result of resonance of a zero point field on the Kerr ring, in the spirit of the semiclassical Casimir effect. It is remarkable, that similar to the quantum case, such excitations of the Kerr string take place without damping since outgoing radiation is compensated by ingoing one. Due to the twofolded topology, the Kerr circular ring turns out to be the "Alice" string, which corresponds to a very minimal nonabelian generalization of the Einstein-Maxwell system. A truncation of the negative sheet leads to the appearance of a source in the form of a relativistically rotating disk [2] and to the class of the superconducting disklike [5] and baglike [9] models of spinning particle.

Appendix B: Axial stringy system

Let us consider solutions for traveling waves - electromagnetic excitations of the Kerr circular string. The problem of electromagnetic excitations of the Kerr black hole has been intensively studied as a problem of the quasinormal modes. However, compatibility with the holomorphic structure of the Kerr space-time put an extra demand on the solutions to be aligned to the Kerr PNC, which takes the form $F^{\mu\nu}k_\mu = 0$. The aligned wave solutions for electromagnetic fields on the Kerr-Schild background were obtained in the Kerr-Schild formalism [22]. We describe here only the result referring for details to the papers [12, 13]. Similar to the stationary case [22] the general aligned solution is described by two self-dual tetrad components $\mathcal{F}_{12} = AZ^2$ and $\mathcal{F}_{31} = \gamma Z - (AZ)_{,1}$, where function A has the form

$$A = \psi(Y,\tau)/P^2, \tag{4}$$

$P = 2^{-1/2}(1 + Y\bar{Y})$, and ψ is an arbitrary holomorphic function of τ which is a complex retarded-time parameter. Function $Y(x) = e^{i\phi}\tan\frac{\theta}{2}$ is a projection of sphere on a complex plane. It is singular at $\theta = \pi$, and one sees that such a singularity will be present in any holomorphic function $\psi(Y)$. Therefore, *all the aligned e.m. solutions turn out to be singular at some angular direction θ.* The simplest modes

$$\psi_n = qY^n \exp i\omega_n\tau \equiv q(\tan\frac{\theta}{2})^n \exp i(n\phi + \omega_n\tau) \tag{5}$$

can be numbered by index $n = \pm 1, \pm 2, ...$, which corresponds to the number of the wave lengths along the Kerr ring. The mode $n = 0$ is the Kerr-Newman stationary field. Near the positive z^+ semi-axis we have $Y \to 0$ and near the negative z^- semi-axis $Y \to \infty$.

Omitting the longitudinal components and the radiation field γ one can obtain [12, 13] the form of the leading wave terms

$$\mathcal{F}|_{wave} = f_R\, d\zeta \wedge du + f_L\, d\bar{\zeta} \wedge dv, \tag{6}$$

where $f_R = (AZ)_{,1}$; $\qquad f_L = 2Y\psi(Z/P)^2 + Y^2(AZ)_{,1}$ are the factors describing the "left" and "right" waves propagating along the z^- and z^+ semi-axis correspondingly.

The behavior of function $Z = P/(r + ia\cos\theta)$ determines a singularity of the waves at the Kerr ring, so the singular waves along the ring induce, via function Y, singularities at the $z^\pm$ semi-axis. We are interested in the asymptotical properties of these singularities. Near the z^+ axis $|Y| \to 0$, and by $r \to \infty$, we have $Y \simeq e^{i\phi}\frac{\rho}{2r}$ where ρ is the distance from the z^+ axis. Similar, near the z^- axis $Y \simeq e^{i\phi}\frac{2r}{\rho}$ and $|Y| \to \infty$. The parameter $\tau = t - r - ia\cos\theta$ takes near the z-axis the values $\tau_+ = \tau|_{z^+} = t - z - ia, \quad \tau_- = \tau|_{z^-} = t + z + ia.$

For $|n| > 1$ the solutions contain the axial singularities which do not fall of asymptotically, but are increasing that means instability. Therefore, only the wave solutions with $n = \pm 1$ turn out to be admissible. The leading singular wave for $n = 1$,

$$\mathcal{F}_1^- = \frac{4qe^{i2\phi+i\omega_1\tau_-}}{\rho^2}\, d\bar\zeta \wedge dv, \tag{7}$$

propagates to $z = -\infty$ along the z^- semi-axis.

The leading wave for $n = -1$,

$$\mathcal{F}_{-1}^+ = -\frac{4qe^{-i2\phi+i\omega_{-1}\tau_+}}{\rho^2}\, d\zeta \wedge du, \tag{8}$$

is singular at z^+ semi-axis and propagates to $z = +\infty$. The described singular waves can also be obtained from the potential $\mathcal{A}^\mu = -\psi(Y,\tau)(Z/P)k^\mu$. The $n = \pm 1$ partial solutions $\mathcal{A}_n^\pm$ represent asymptotically the singular plane-fronted e.m. waves propagating along z^+ or z^- semi-axis without damping. The corresponding self-consistent solution of the Einstein-Maxwell field equations are described in [12]. They are singular plane-fronted waves having the Kerr-Schild form of metric () with a constant vector k^μ. For example, the wave propagating along the z^+ axis has $k^\mu dx^\mu = -2^{1/2} du$). The Maxwell equations take the form $\Box \mathcal{A} = J = 0$, where $\Box$ is a flat D'Alembertian, and can easily be integrated leading to the solutions $\mathcal{A}^+ = [\Phi^+(\zeta) + \Phi^-(\bar\zeta)] f^+(u) du, \qquad \mathcal{A}^- = [\Phi^+(\zeta) + \Phi^-(\bar\zeta)] f^-(v) dv$, where $\Phi^\pm$ are arbitrary analytic functions, and functions $f^\pm$ describe the arbitrary retarded and advanced waves. Therefore, the wave excitations of the Kerr ring lead to the appearance of singular pp-waves which propagate outward along the z^+ and/or z^- semi-axis.

These axial singular strings are evidences of the axial stringy currents, which are exhibited explicitly when the singularities are regularized. Generalizing the field model to the Witten field model for the cosmic superconducting strings [24], one can show [13] that these singularities are replaced by the chiral superconducting strings, formed by a condensate of the Higgs field, so the resulting currents on the strings are matched with the external gauge field.

The stringy system containing the chiral modes of only one direction cannot exist since it is degenerated in a word-line [11]. A combination of two $n = \pm 1$ excitations leads to the appearance of two semi-infinite singular D-strings of opposite chirality as it is shown at the fig.1. Similar to the all ray of PNC, the semi-infinite singularities can be extended to the negative sheet passing through the Kerr ring. The world-sheet of such a system acquires the structure of an orientifold and is given by

$$x^\mu(t, z) = \frac{1}{2}[(t - z) k_R^\mu + (t + z) k_L^\mu], \qquad (9)$$

where the lightlike vectors k^μ are constant and normalized. At the rest frame the time-like components are equal $k_R^0 = k_L^0 = 1$, and the space-like components are oppositely directed, $k_R^a + k_L^a = 0, \quad a = 1,2,3$. Therefore, $\dot{x}^\mu = (1,0,0,0)$, and $x'^\mu = (0, k^a)$, and the Nambu-Goto string action $S = \alpha'^{-1} \int\int \sqrt{(\dot{x})^2 (x')^2 - (\dot{x} x')^2} dt dz$ can be expressed via k_R^μ and k_L^μ.

For the system of two D-strings in the rest one can use the gauge with $\dot{x}^0 = 1, \quad \dot{x}^a = 0$, where the term $(\dot{x} x')^2$ drops out, and the action takes the form $S = \alpha'^{-1} \int dt \int \sqrt{p^a p_a} d\sigma$, where $p^a = \partial_\sigma x^a = \frac{1}{2}[x'^\mu_R(t+\sigma) - x'^\mu_L(t-\sigma)]$.

To normalize the infinite string we have to perform a renormalization putting $m = \alpha'^{-1} \int (x')^2 dz$, which yields the usual action for the center of mass of a pointlike particle $S = m \int \sqrt{(\dot{x})^2} dt$.

Appendix C: Orientifold and a complex Kerr string

The tension for a free particle tends to zero, but it can be finite for the bounded states when the axial string forms the closed loops. An extra tension can appear in the bounded systems, since an extra magnetic flow can concentrate on the closed loops. By formation of the closed loops chiral modes have to be matched forming an orientifold structure. The position x^μ of the (say)'left' semi-string, being extended to the 'negative' sheet, has to coincide with the position of the 'right' one. For a free stationary particle it is realized. However, in general case it puts a very strong restriction on the dynamics of the string. The orientifold of the axial string is also suggested by the structure of superconducting strings. As it pointed out by Witten [24], it contains the light-like fermions of both the chiralities, moving in opposite directions, so the massive Dirac solutions for a superconducting string describe an interplay of the trapped fermions of the opposite chiralities. In general case orientifolding the world-sheet requires a very high degree of symmetry for the string. It turns out that this is provided by the holomorphy and orientifold structure of the third, *complex string* of the Kerr geometry [7].

In many aspects this string is similar to the $N = 2$ string [21, 26], but it has the signature $(- + ++)$ and the euclidean world sheet.

It was obtained in [7, 11] that boundary conditions of the complex string demand the orientifold structure of the world-sheet. The resulting field equations have the solutions which are satisfied by the holomorphic (!) modes $X_0^\mu(\tau)$. These solutions are the 'left' modes, and the 'right' modes appear by orientifolding. The interval for parameter $\sigma = a\cos\theta$ is doubled: $\Sigma_\pm = [-a, a]$, forming a circle $S^1 = \Sigma_+ \cup \Sigma_-$ which is parametrized by $\theta \in [0, 2\pi]$, so Σ_- parametrizes the string in opposite orientation. Orientifold is formed by a Z_2 factorization of the world-sheet. The string turns into a closed but folded one. The 'right' modes have the form of the 'left' ones but have a support on the reversed interval $\theta \in [\pi, 2\pi] \in \Sigma_-$:

$$X^\mu_{0R}(t + ia\cos(2\pi - \theta)) = X^\mu_{0L}(t + ia\cos\theta), \tag{10}$$

which is a well-known kind of extrapolation [26]. So, for the period $(0, 2\pi)$ occurs a flip of the modes $left \to right$. It is accompanied by a space reversal of the twistors joined to the points of the world-sheet, which is performed by the exchange $Y \to -1/\bar{Y}$. As a result the orientifold structure of the complex string provides the orientifold structure for the chiral axial strings.

It can be shown that all the three orientifold structures can be joined forming a unite orientifold matched to the holomorphic twistorial structure of the Kerr space-time. We intend to discuss it in details elsewhere.

Since the Kerr spinning particle is massive, its momentum along the z-axis is represented as a sum of the lightlike parts $p^\mu = p^\mu_L + p^\mu_R$, where the corresponding spinors are

$$p^\alpha_L =< p_L| = p_L \cdot < k_L| \tag{11}$$

and

$$p^\alpha_R =< p_R| = p_R \cdot < k_R|. \tag{12}$$

For a relativistic motion we have either $p_L << p_R$ or $p_L >> p_R$, which determines the sign of helicity, and as a result one of the axial semi-strings turns out to be strongly dominant for the scattering. It may justify the use of the very reduced description of the Kerr spinning particle via the spinor helicity formalism. We are led to the conclusion that the axial stringy system may be responsible for the high energy scattering processes.

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