APPROACH UNIFYING SPINS AND CHARGES

or

From one Weyl spinor and only the gravitational gauge field of $SO(q, d-q)$ in $d \geq 1+13$

to

families of quarks and leptons of all the known charges and massess and to known fields in $d=1+3$

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I am proposing **the approach [1,2] that unifies all the internal degrees of freedom, that is the spin and all the known charges** in $d = 4$, to only the spin in $d \geq 14$ and accordingly also all the gauge fields to only the gravity. The approach is a kind of Kaluza-Klein-like theories.

The approach is answering some of the open questions of the Standard model like: i) where do families come from, what could be the origin of the Yukawa couplings and do we really have only three families at low energies (we predict four), ii) why does the weak charge breaks parity, while the colour does not and why do only left handed quarks and leptons carry the weak charge while the right ones do not, iii) how the break o symmetries could occur to come to observable charges and almost massless fermions. iv) We also show on an example that there exist at list one case in which the Witten's no-go theorem for massless spinors in subspaces is avoided.

d ordinary and d Grassmann coordinates are assumed

$$d \ldots \quad x^a \qquad \text{and} \qquad d \ldots \quad \theta^a$$

$$\{x^a, x^b\}_- = 0, \qquad \{\theta^a, \theta^b\}_+ = 0,$$

$$p^{xa} = i \overrightarrow{\partial}_{xa}, \qquad p^{\theta}_{\theta a} = -i \overrightarrow{\partial}_{\theta a},$$

$$\{x^a, p^{xb}\}_- = -i\eta^{ab}, \qquad \{\theta^a, p^{\theta b}\}_+ = -i\eta^{ab}.$$

It can be demonstrated [3] that the internal space of spins and charges can equally well be described by either the Grassmann coordinates, or by the differential forms or by the Clifford algebra objects [4,6]

$$\gamma^a : = i(p^\theta_{\theta a} - i\theta^a),$$
$$\tilde{\gamma}^a : = (p^\theta_{\theta a} + i\theta^a).$$

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}, \quad \{\gamma^a, \tilde{\gamma}^b\} = 0$$

$$S^{ab} = \frac{i}{4}[\gamma^a, \gamma^b], \quad \tilde{S}^{ab} = \frac{i}{4}[\tilde{\gamma}^a, \tilde{\gamma}^b], \quad [S^{ab}, \tilde{S}^{cd}] = 0$$

Basis in the internal part of the Hilbert space can be presented as polynomials of either θ^a or γ^a, respectively or differential forms

$$< \theta^a|\Phi > = \sum_{i=0,.,d} \sum_{\{a_1<.<a_i\}\in\{0,.,d\}} \alpha_{a_1,a_2,\dots,a_i}\theta^{a_1}\theta^{a_2}\cdots\theta^{a_i},$$

or

$$< \gamma^a|\Phi > = \sum_{i=0,.,d} \sum_{\{a_1<.<a_i\}\in\{0,.,d\}} \alpha_{a_1,a_2,\dots,a_i}\gamma^{a_1}\gamma^{a_2}\cdots\gamma^{a_i}.$$

One can arranged any of these basis to be the eigenbasis of the Cartan subalgebra (of $d/2$ commuting S^{ab}). It turns out that it is comfortable to work with the Clifford objects only, using the technique presented in ref. [4,6].

$$\stackrel{ab}{(\pm i)}:= \frac{1}{2}(\gamma^a \mp \gamma^b), \;\; \stackrel{ab}{[\pm]}:= \frac{1}{2}(1 \mp \gamma^a\gamma^b),$$

if a or b is a time

$$\stackrel{ab}{(\pm)}:= \frac{1}{2}(\gamma^a \pm i\gamma^b), \;\; \stackrel{ab}{[\pm]}:= \frac{1}{2}(1 \pm i\gamma^a\gamma^b),$$

if a and b are space like

$$S^{ab} \stackrel{ab}{(\pm i)} = \pm\frac{i}{2} \stackrel{ab}{(\pm i)}, \;\; S^{ab} \stackrel{ab}{[\pm i]} = \pm\frac{i}{2} \stackrel{ab}{[\pm i]}$$

$$S^{ab} \stackrel{ab}{(\pm)} = \pm\frac{1}{2} \stackrel{ab}{(\pm)}, \;\; S^{ab} \stackrel{ab}{[\pm]} = \pm\frac{1}{2} \stackrel{ab}{[\pm]}\,.$$

One state of one irreducible representation of

$SO(1,13)$ can accordingly be graphically represented as

$$\begin{array}{ccccccccc} \overset{03}{(+i)}\overset{12}{(+)} & | & \overset{56}{(+)} & \overset{78}{(+)} & || & \overset{9\,10}{(+)} & \overset{11\,12}{(-)} & \overset{13\,14}{(-)} & |\psi\rangle \end{array}$$

$$SO(1,3) \quad SU(2) \qquad SU(3)$$

$|\psi\rangle$ is a vacuum state. The state above represents a right handed u quark with spin up and of the colour $(1/2, 1/(2\sqrt{3})$. All the other states of the Weyl spinor follow from the starting state by the application of the generators of the Lorentz transformations. All are eigenstates of the Casimir

$$\Gamma^{(d)} := (i)^{d/2} \prod_a (\sqrt{\eta^{aa}}\gamma^a), \quad \{S^{ab}, \Gamma^{(d)}\}_- = 0.$$

For the above one Weyl spinor the egenvalue of $\Gamma^{(d)}$ is -1.

We demonstrate

i) that **one** (only one!) **Weyl left handed multiplet** if represented in a way to demonstrate the $SU(3), SU(2)$ and $U(1)$ substructure of the group $SO(1,13)$ (with $2^{(14/2-1)}$ members) **contains the spinors** (the quarks and the leptons) **AND the "antispinors"**(the antiquarks and antileptons) of **the Standard model** (with the right handed weak chargeless neutrino and the left handed weak chargeless antineutrino in addition) (TABLE 1),

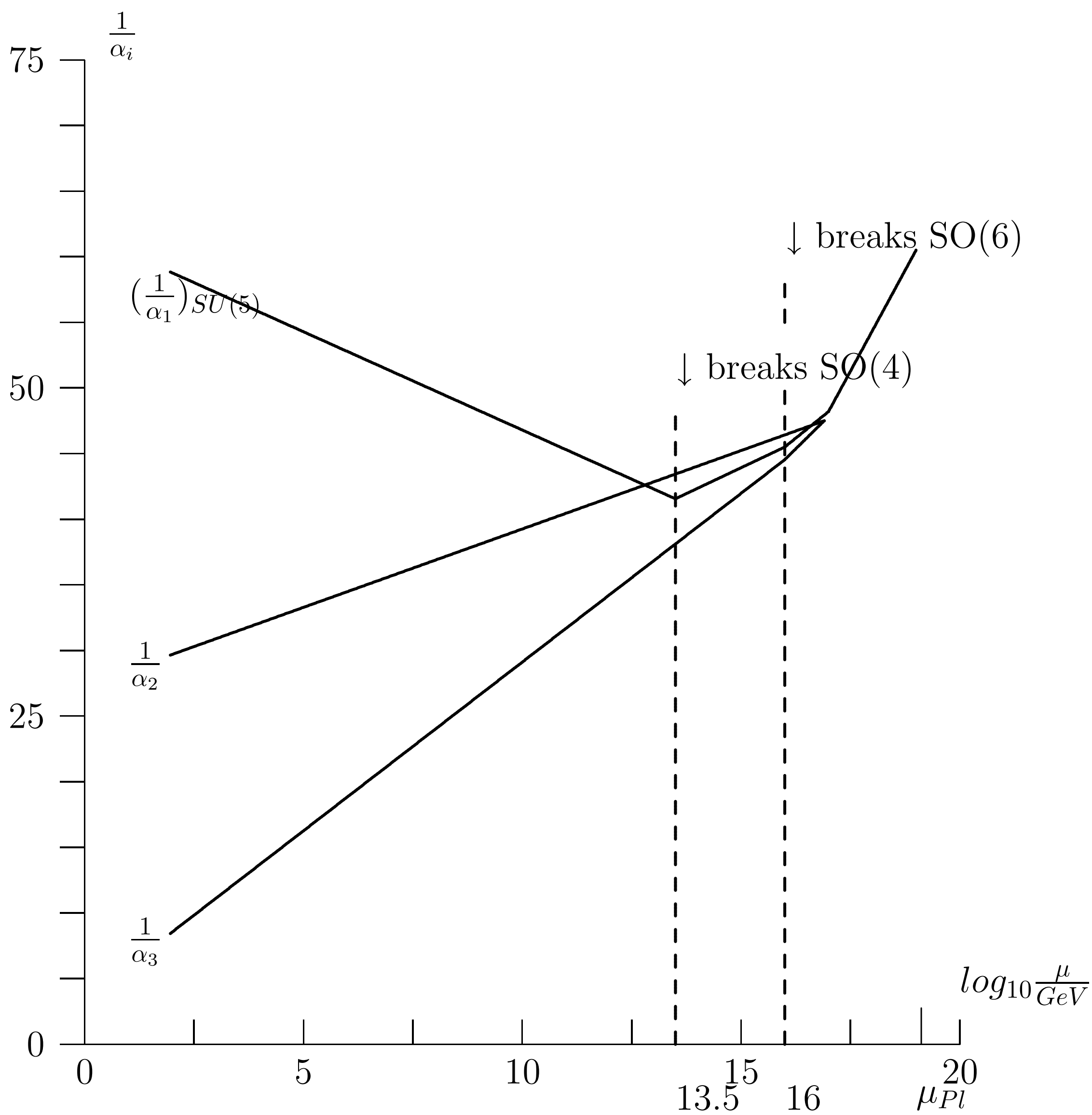
ii) that one easily understands in this presentation **why the weak charge breaks parity while the colour charge does not** (TABLE 2),

iii) that **there are several possible ways**

of breaking the group $SO(1,13)$, like

$$
\begin{array}{c}
\underbrace{\begin{array}{ccc}
 & \mathrm{SO}(1,13) & \\
 & \downarrow & \\
 & \mathrm{SO}(1,7)\otimes\mathrm{SO}(6) & \\
 \swarrow & & \searrow \\
 \mathrm{SO}(1,7) & & \mathrm{SU}(4) \\
 \swarrow & & \downarrow \\
 \mathrm{SO}(1,3)\otimes\mathrm{SO}(4) & & \mathrm{SU}(3)\otimes\mathrm{U}(1) \\
 \downarrow & & \downarrow \\
 \mathrm{SO}(1,3)\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1) & & \mathrm{SU}(3)\otimes\mathrm{U}(1)
\end{array}} \\
\mathrm{SO}(1,3)\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1)\otimes\mathrm{SU}(3)\otimes\mathrm{U}(1) \\
\downarrow \\
\mathrm{SO}(1,3)\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1)\otimes\mathrm{SU}(3)
\end{array}
$$

leading to the known spins, charges and flavors of the leptons and the quarks and the anti-leptons and the anti-quarks, guaranteeing unification of the coupling constants (including the appearance of the p decay at the large enough scale)



We found the running coupling constants, extrapolated from the experimental values by the assumption that the gauge group $SO(4)$ breaks at much lower scale (at around 10^{13} GeV) than the gauge group $SO(6)$ (which breaks at around 10^{17} GeV).

The $SU(2)$ gauge group coupling constant does not change when running together with $(\alpha_{U(1),SO(4)})^{-1}$. The three α's meet and then run together as $SO(1,13)$ (rather SO(10)).

iv) that **there is the mechanism for generating** *families* **of quarks and leptons**. Namely, while S^{ab} transform the starting state into all the states of one Weyl irreducible representation

$$S^{01}\; \stackrel{03}{(+i)}\stackrel{12}{(+)}\;|\;\stackrel{56}{(+)}\stackrel{78}{(+)}\;||\;\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(+)}\;\;\stackrel{13\,14}{(+)} = \stackrel{03}{[-i]}\stackrel{12}{[-]}\;|\;\stackrel{56}{(+)}\stackrel{78}{(+)}\;||\;\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(+)}\;\;\stackrel{13\,14}{(+)}\;.$$

$\tilde{S}^{ab}$ transform a starting state of one Weyl spinor into a starting state of another Weyl spinor- *to another family.*

$$\tilde{S}^{01}\; \stackrel{03}{(+i)}\stackrel{12}{(+)}\;|\;\stackrel{56}{(+)}\stackrel{78}{(+)}\;||\;\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(+)}\;\;\stackrel{13\,14}{(+)} = \stackrel{03}{[+i]}\stackrel{12}{[+]}\;|\;\stackrel{56}{(+)}\stackrel{78}{(+)}\;||\;\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(+)}\;\;\stackrel{13\,14}{(+)}\;.$$

We demonstrate that **some components of spin connections and vielbeins** might **manifest as Yukawa couplings**. The *Lagrange function for a Weyl spinor, interacting with only the gravitational field in d dimensional space*:

$$\mathcal{L} = \bar{\psi}\gamma^a p_{0a}\psi = \bar{\psi}\gamma^a f^{\mu}_{a} p_{0\mu},$$

$$\text{with} \quad p_{0\mu} = p_\mu - \frac{1}{2}S^{ab}\omega_{ab\mu} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\mu}.$$

$$\begin{aligned}\mathcal{L} = &\bar{\psi}\gamma^\alpha(p_\alpha - \sum_{A,i} g^A \tau^{Ai} A^{Ai}_\alpha \psi) \\ &+ i\psi^+ S^{0h} S^{kk'} f^\sigma_h \omega_{kk'\sigma}\psi \\ &+ \quad i\psi^+ S^{0h} \tilde{S}^{kk'} f^\sigma_h \tilde{\omega}_{kk'\sigma}\psi,\end{aligned}$$

manifests in the "physical" part of space all the needed degrees of freedom of spinors - *the* *spin and all the known charges* (TABLE 1).

The gravitational field (spin connections and vielbeins) *manifests accordingly* in 4-dimensional part of space as all the known gauge fields as well as *the Yukawa couplings*, if the break of symmetries occurs in an appropriate way.

The 2^{nd} and the 3^{rd} terms are mass term, $f^{\sigma}_{h}\tilde{\omega}_{kk'\sigma}$ behaves in $d(=1+3)-$ dimensional subspace like a scalar field, while $S^{0h}, h = 7, 8,$ for example, transforms a right handed weak chargeless spinor into a left handed weak charged spinor, without changing the spin.

The term

$$\psi^{+} S^{0h} S^{kk'} f^{\sigma}_{h} \omega_{kk'\sigma} \psi = -\gamma^{0}\gamma^{h}\tau^{Ai} A^{Ai}_{\sigma} f^{\sigma}_{h}$$

and the term, connecting different families

$$i\psi^{+} S^{0h} \tilde{S}^{kk'} f^{\sigma}_{h} \tilde{\omega}_{kk'\sigma} \psi$$

represent the Yukawa couplings.

v) that **the approach suggests symmetries of the mass matrices and argues for the appearance of the fourth family**, with all the properties (besides the masses) of the three known families (all in agreement with Okun at al). An appropriate break of symmetries leads to the following four families

$$\stackrel{03}{(+i)}\stackrel{12}{(+)}\stackrel{56}{(+)}\stackrel{78}{(+)}$$

$$\stackrel{03}{[+i]}\stackrel{12}{[+]}\stackrel{56}{(+)}\stackrel{78}{(+)}$$

$$\stackrel{03}{(+i)}\stackrel{12}{(+)}\stackrel{56}{[+]}\stackrel{78}{[+]}$$

$$\stackrel{03}{[+i]}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{[+]}\;.$$

The above suggestion for the four families of quarks and leptons leads to a mass matrix of the form

$$\begin{pmatrix} A_a & B_a & C_a & D_a \\ B_a & C_a & D_a & E_a \\ C_a & D_a & A_a & B_a \\ D_a & E_a & B_a & F_a \end{pmatrix},$$

a is u, d, ν, e.

The mass matrices have the symmetry

$$\begin{pmatrix} X & Y \\ Y & X' \end{pmatrix},$$

and **predicts acceptable masses and angles of the "CKM" matrix for the four generations**.

vi) that the approach predicts **small charges of the observed spinors** (and “antispinors“) and the **anomaly cancellation**.

vii) that we [7,9] **have shown on an example of $M^{(1+5)}$, compactified to** $M^{(1+3)} \times M^{(2)}$ that there are **massless** spinors in $M^{(1+3)}$ **chirally coupled** with the Kaluza-Klein gauge charge to the corresponding gauge field, which originates in **gravity with only spin connections and vielbeins**, as assumed also in the proposed approach.

vii) We show that there are the internal degrees of freedom - the spin degree of freedom - which requires even dimensional spaces and an odd number of time coordinates.

viii) We show how the equations of motion and the inner product looks like in spaces with more than one time coordinate.

Conclusions:

The **approach, unifying spins and charges**, was presented, which demonstrates that that:

a) A **left handed** $SO(1,13)$ (rank 7) **Weyl multiplet** (no mirror symmetry) **contains**, if demonstrating the $SO(1,3)$, $SU(2)$ and $U(1)$'s substructure of the group $SO(1,13)$, just **all the quarks and the leptons and all the anti-quarks and the anti-leptons of the Standard model**.

b) In the proposed approach the concept of **families naturally appear**s, the families are reached from the starting family by the operators $\tilde{S}^{ab} = -i/4[\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a]$.

c) We propose a Lagrange density for spinors in $d = 14$ dimensional space

$$\psi^+ \gamma^0 \gamma^a f^{\mu}_{a} \, (p_\mu - \frac{1}{2} \mathcal{S}^{bc} \, \omega_{bc\mu} \,) \, \psi,$$

with **spin connections** $\omega_{bc\mu}$ **and vielbeins** f^{μ}_{a}**,** which then manifest at low energies as **all the known gauge fields** A^{Ai}_{α}

$$\psi^+ \gamma^0 \gamma^m f^{\alpha}_{m} \, (p_\alpha - \sum_{Ai} \, \tau^{Ai} A^{Ai}_{\alpha}) \psi$$

as well as **the Yukawa couplings**

$$\psi^+ \gamma^0 \gamma^h S^{h'h''} \omega_{h'h''\sigma} f^{\sigma}_{h} \psi,$$

$$\psi^+\gamma^0\gamma^h\tilde{S}^{h'h''}\tilde{\omega}_{h'h''\sigma}f^\sigma_h\psi.$$

Both types of terms manifest accordingly as mass terms of quarks and leptons. It is the interactions in higher dimensions which look like a Higgs causing the Yukawa couplings in a four-dimensional part of space.

d) The mass matrices, suggested by the approach, predict **four families of quarks and leptons** and the corresponding "CKM" matrix.

e) We presented a possible **scheme of breaking symmetry** $SO(1,13)$, leading to **unification of the coupling constants** and the acceptable prediction for the p decay.

f) We are able to **prove on at least one example**, that the **genuine Kaluza-Klein**-like theories, as it is also the proposed approach, can **lead to massless spinors in the "physical" part of space, chirally coupled** to the correponding gauge field.

h) The approach initiated the **technique for representing spinors** in an elegant way.

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